Calculus 1 -- Workshop 1: An Introduction to Calculus

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References:
Smith and Minton, **Calculus, 2nd edition**, Section 0.8, pp. 72-74

http://www.recordball.com/

John Bain has created this gigantic rubber band ball from 850,000 rubber bands. It is approximately 5' in diameter and weighs over 3,000 lbs. He started with handfuls of elastics that were free from the post office, and the ball kept growing!

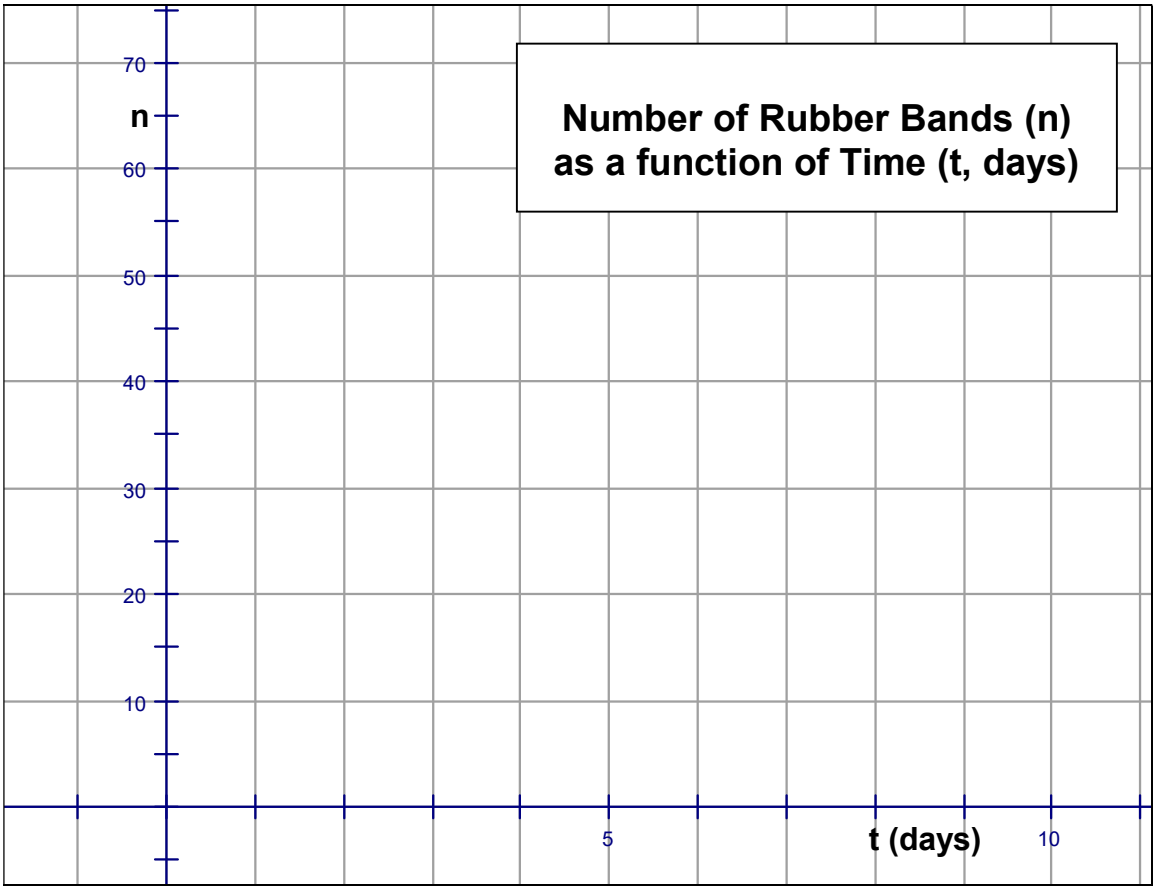
PART I

1. Inspired by John Bain's story, you decide to construct your own rubber band ball. Every day you add 5 rubber bands to the ball. What is the rate of change (in rubber bands per day) of the number of rubber bands on the ball?

2. If you start with zero rubber bands, complete the following table for days 1 – 10:

Day	Rate at which the Rubber Band Ball is Growing (rubber bands/ day)	Number of Rubber Bands in the Ball
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

3. On the graph below, plot the data points (day, number of rubber bands) from the previous table.



4. A smooth curve drawn through these data points would have what shape?

5. What is the slope of the curve? What does it represent?

You decide to abandon your rubber band ball effort, and begin building a paper clip chain. You start with 10 paper clips on your chain. In the first week you add 3 paper clips to the chain. During the second week you add 6 paper clips to the chain. During the third week you add 9 paper clips to the chain. Each week the number of paper clips you add to the chain is three more than the number you added the week before.

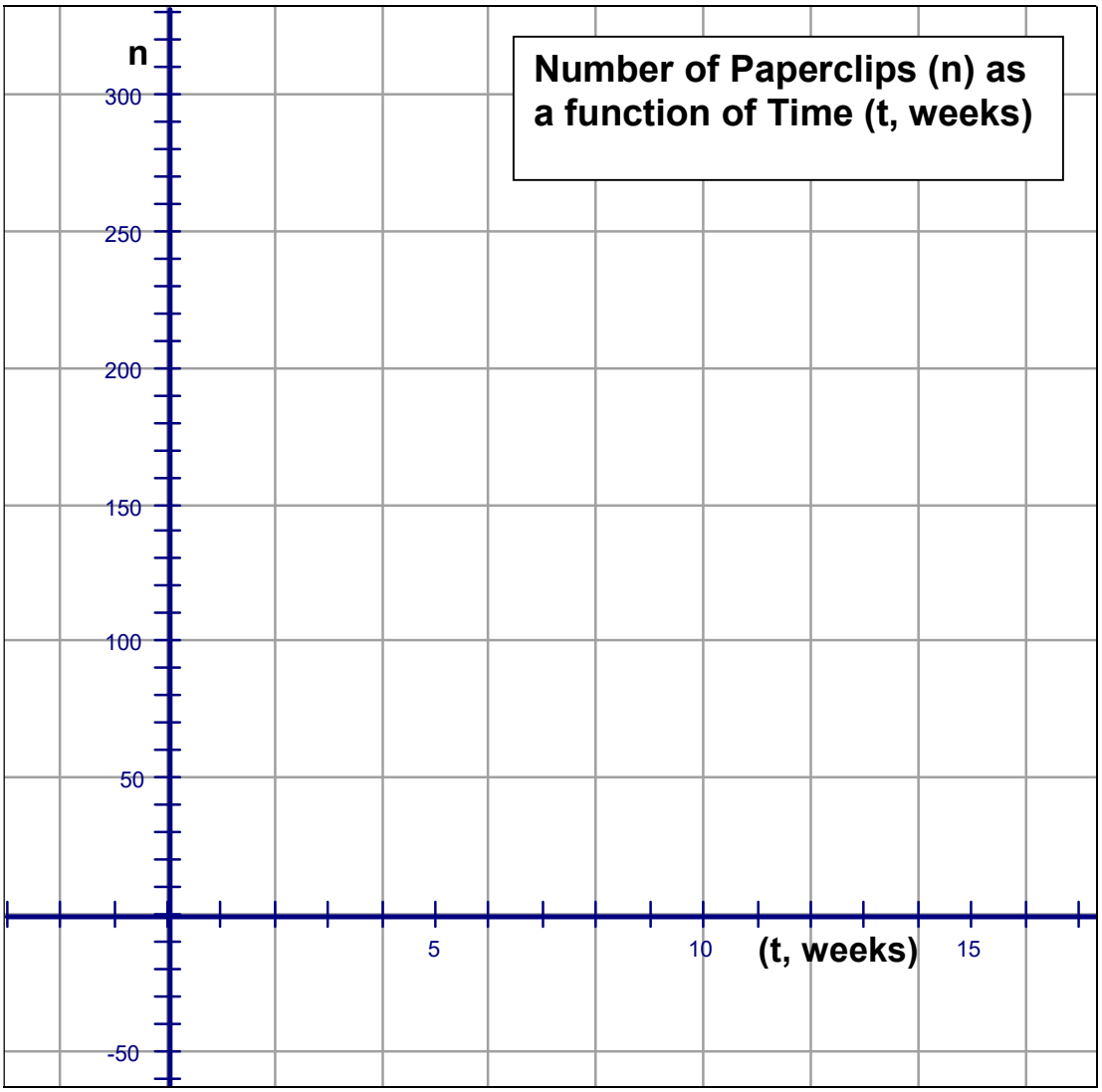
6. Complete the following table for weeks 0-10:

Week	**Rate at which the Paper Clip Chain is Growing (paper clips/ week)**	**Number of Paper Clips in the Chain**
0	**-**	**10**
1	3	
2	6	
3	9	
4		
5		
6		
7		
8		
9		
10		

7. What is the rate of change (in paper clips per week) of the number of paper clips on the chain? What makes this question more difficult to answer than question 1?

8. If you could not answer the general question in question 7, how about this one: In the tenth week what is the rate of change (in paper clips per week) of the number of paper clips on the chain?

9. Plot the data points (week, number of paperclips) from the table in question 6 on the graph below, and draw a smoothed curve through the data points



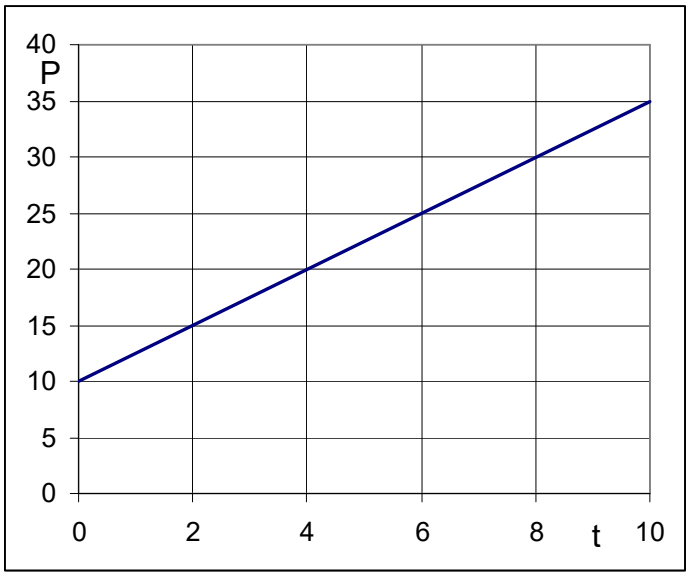
10. How does this graph differ from the one for the rubber bands?

11. How is this difference related to the rates of change of the two quantities?

12. How would you describe the slope of the curve?

13. The graph of the equation $P = 2.5t + 10$ is shown at the right.

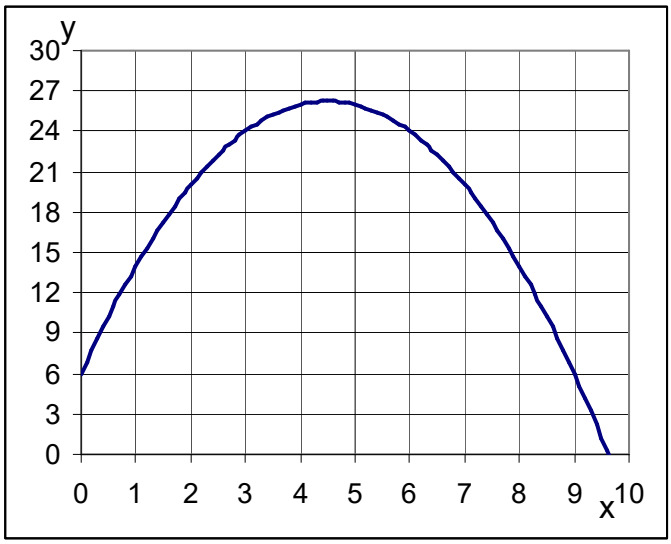
What is the rate of change of P with respect to t?



14. The graph of the equation $y = -x^2 + 9x + 6$ is shown at the right.

Consider the question: What is the rate of change of y with respect to x?

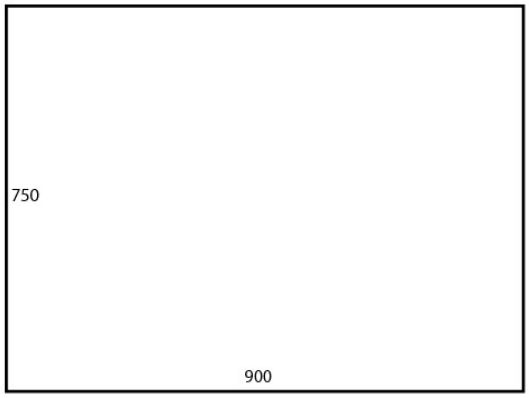
Can you answer it? If yes, what is the answer; if no, what causes difficulties in trying to answer it?



15. Instead consider the question: What is the rate of change of y with respect to x when x = 2? Discuss how you could answer that using the graph provided above. Try to answer it if you can.

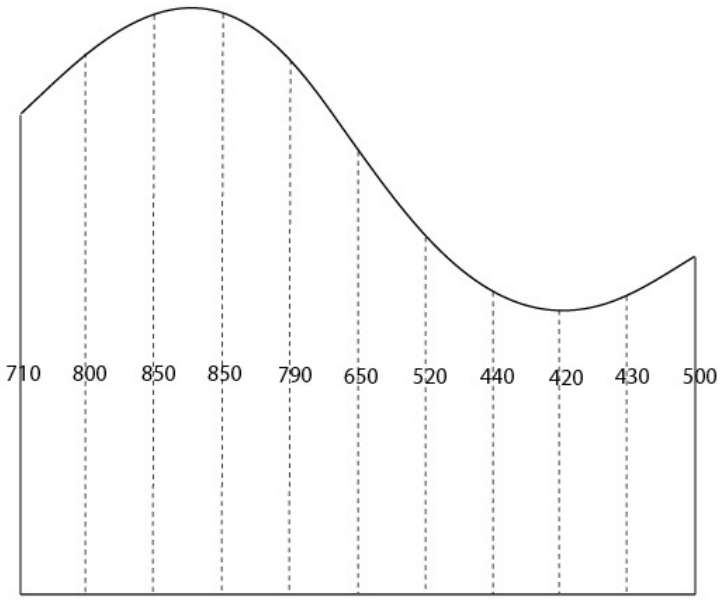
PART II

16. A farmer has a field as shown at the right. The measurements are in feet. What is the area of the farmer's field?



17. A farmer has a field as shown at the right. One side is bordered by a river, and the side opposite the river is bordered by a road. The other two bordering sides of the field run perpendicular to the road, from the road to the river. The farmer wishes to approximate the area of her field, so at 100 foot intervals along the road, she measured the distance (in feet) from the road to the river. Her measurements are shown at the right.

Use the information provided to approximate the area of the field, and explain how you arrived at your approximation.

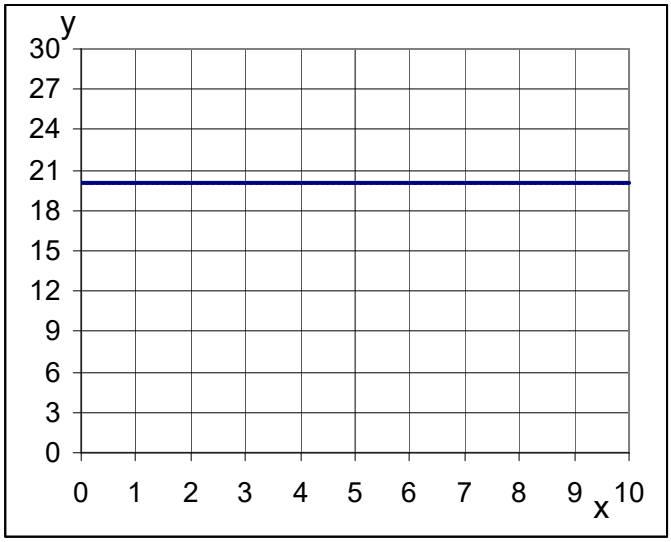


18. Discuss what could be done to make a more accurate approximation of the area.

19. The graph of the equation $y = 20$ is shown at the right.

Shade in the area between $x = 2$ and $x = 9$ that is bounded above by the graph and bounded below by the x axis.

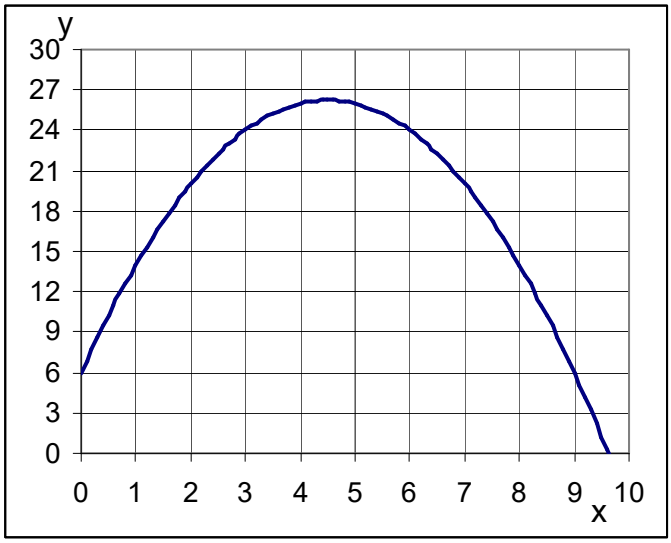
What is the area of the figure you shaded?



20. The graph of the equation $y = -x^2 + 9x + 6$ is shown at the right.

Shade in the area between $x = 2$ and $x = 9$ that is bounded above by the graph and bounded below by the x axis.

Estimate the area of the figure you shaded and discuss how you made your estimate.



21. Discuss how you could improve your estimate.

Conclusion

22. The goal of this workshop was to investigate the two central themes of Calculus. Can you summarize briefly what those ideas are?

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