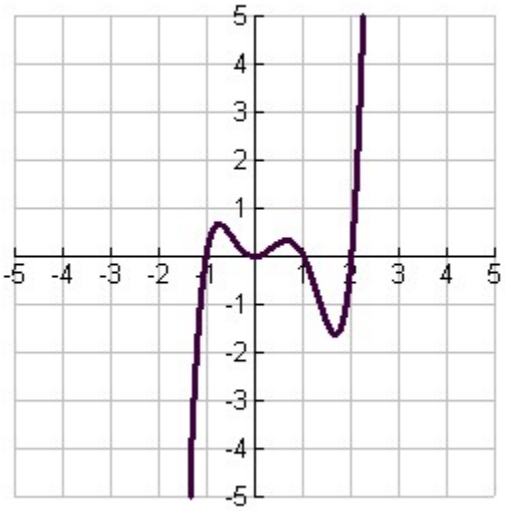


Calculus 1 -- Workshop 3: Relationship between a Function and its Derivative

PAULA DREWNIANY, SUE MCGARRY, JEN TYNE

Taken in part from The Mathematical Association of America's Learning by Discovery, A Lab Manual for Calculus.

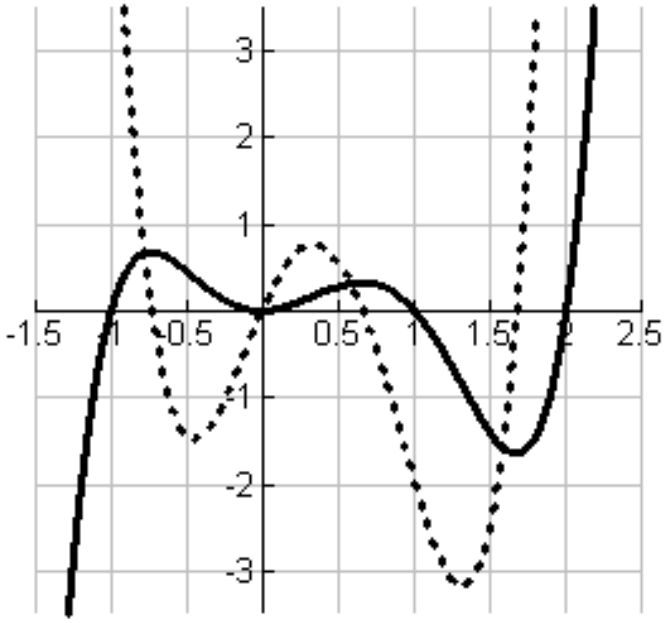
References:
Smith, R. T., Minton, R. B., *Calculus, 2nd edition,* McGraw Hill, 2002, Pages 11-19,24-31, 50-55,168-169, 176-183, and 187-191.

In this workshop, you will be asked to compare the graph of a function (like the one pictured above) to that of its derivative.

The workshop starts on the next page.

The Lab:

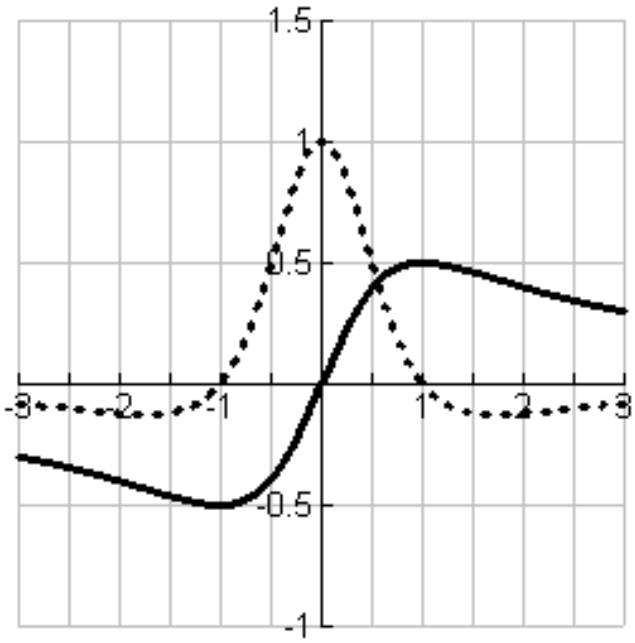
1. Let $f(x) = x^2(x+1)(x-1)(x-2) = x^5 - 2x^4 - x^3 + 2x^2$. Then, $f'(x) = 5x^4 - 8x^3 - 3x^2 + 4x$. Both f and f' are graphed below over the interval [-1.5, 2.5].



   a. Which is the graph of f, and which is the graph of f'? Label the graph

   b. Answer the following questions by inspection of the graph:
      - Over what intervals is f(x) increasing?
      - Over what intervals is f ' (x) > 0?
      - Over what intervals is f(x) decreasing?
      - Over what intervals is f ' (x) < 0?
      - What are the x-coordinates of all the extrema of the graph of f?
      - For what values of x is f ' (x) = 0?

2. Let $f(x) = \frac{x}{1+x^2}$. Then, $f'(x) = \frac{(1+x^2)-2x^2}{(1+x^2)^2}$. Both f and f' are graphed below, over the interval [-3, 3].



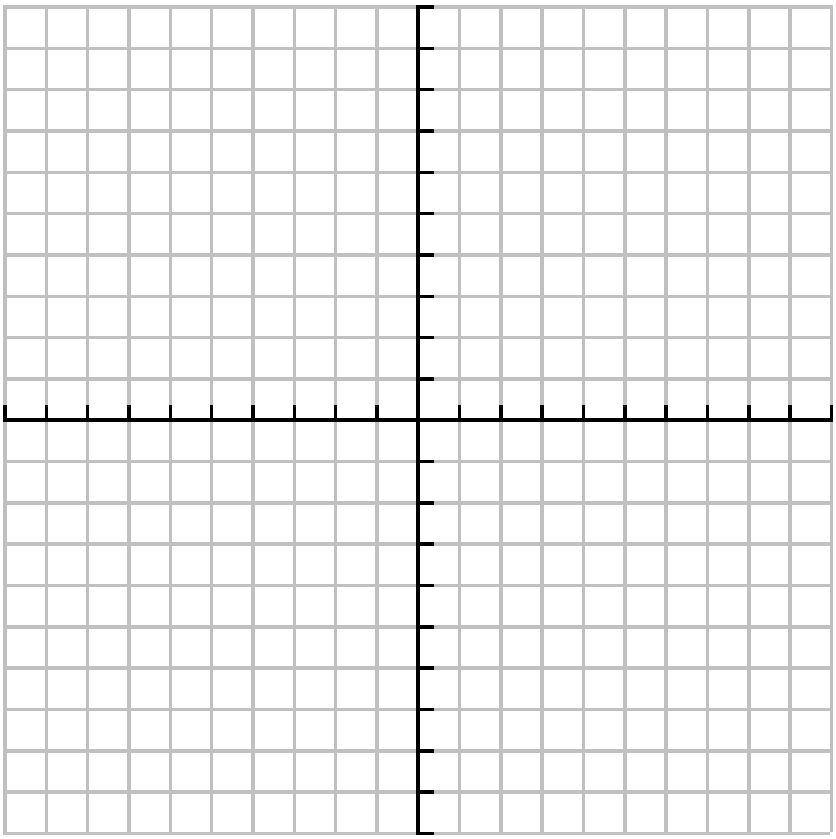
a. Which is the graph of f, and which is the graph of f'? Label the graph.

b. Answer the following questions by inspection of the graph:
   - Over what intervals is f(x) increasing?
   - Over what intervals is f ' (x) > 0?
   - Over what intervals is f(x) decreasing?
   - Over what intervals is f ' (x) < 0?
   - What are the x-coordinates of all the relative extrema of the graph of f?
   - For what values of x is f '(x) = 0?

3. On the basis of your experience so far, write a statement that relates the behavior of a function (where it is increasing, decreasing, has maximum and minimum points) to properties you have observed about the graph of its derivative.

4. Your leader will provide you with a polynomial function and an interval over which to graph it. Graph the function on the interval given.

   a. Draw the graph of y = g(x) below.

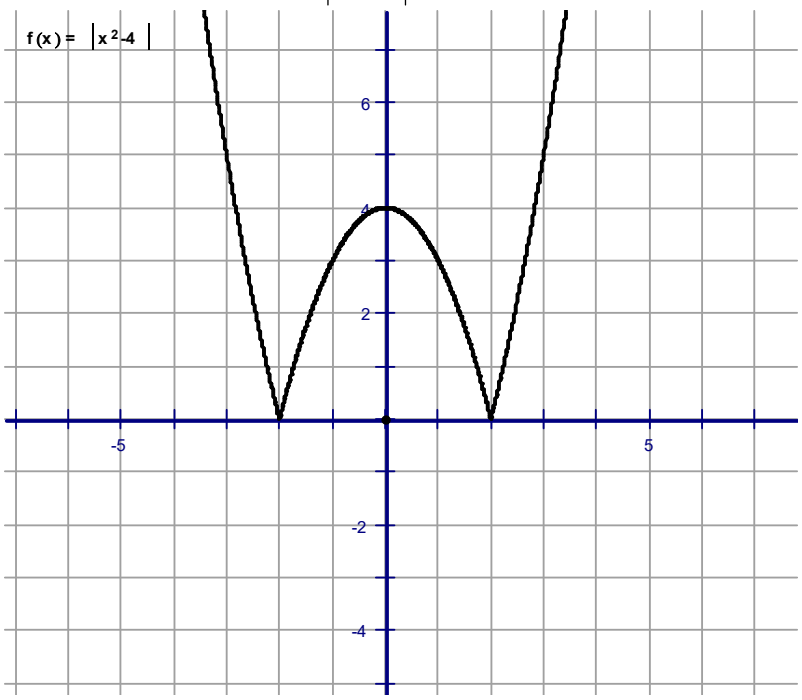


   b. Using your conjecture from #3, imagine the shape of the graph of g'(x). Carefully sketch a graph of g'(x) on the same coordinate axis as g(x) above.

   c. Calculate g'(x) and graph it on your graphing calculator, comparing your sketch with the actual g'(x). How did you do?

5. Consider the function $f(x) = 2^x$. Calculus students often make the **mistake** in calculating the derivative as $f'(x) = x2^{x-1}$. Explain why this cannot be true.

6. Consider the graph of the function $f(x) = |x^2 - 4|$



a. There are two values of x for which the derivative does not exist. What are these values, and why does the derivative not exist there?

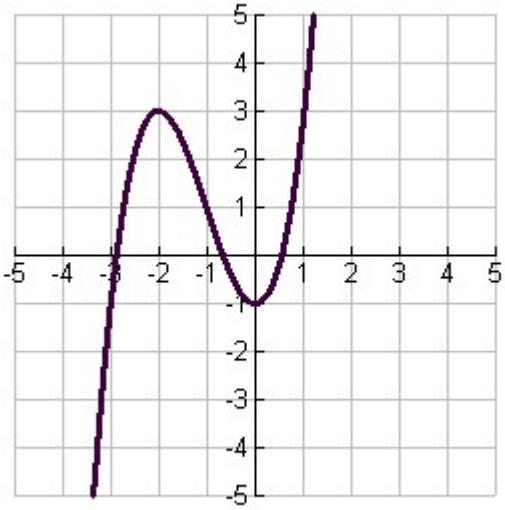
b. To find the derivative of f(x), we first have to define the function in a way that there are no absolute value signs. Define f(x) as a piecewise function with no absolute value sign.

c. Now, compute the derivative separately for each part of your piecewise function.

d. On the graph on page 5, carefully sketch f' (disregarding places where f' is not defined) over the interval [-4, 4].

e. Does your conjecture from #3 still hold? Are any modifications needed?

7. So far, you have worked with the graph of a function to come up with the graph of its derivative. What about going the other way: Can we use the graph of a derivative to come up with the graph of an original function (called an antiderivative)? Suppose you have been given the graph of f ', would you be able to reconstruct the shape of the graph of f? Below is the graph is of f '(x).



a. Use your conjecture to construct a possible graph for the function f(x).

b. Give an explanation of why you graphed it the way you did.

c. Why isn't there a unique function that has f' for its derivative?

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