Calculus 1, Workshop 4: Modeling with Surge Functions –
Nicotine and Blood Alcohol Concentrations

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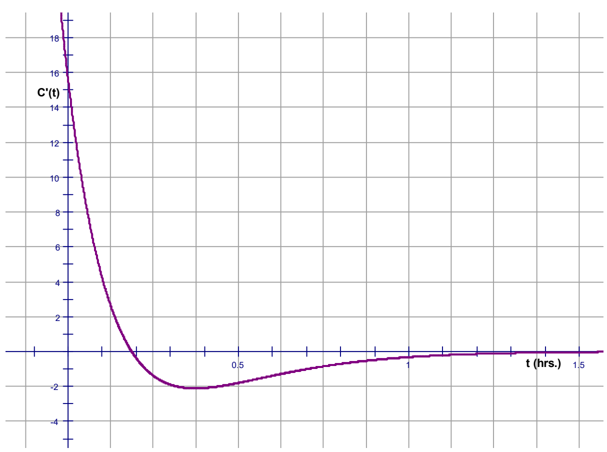
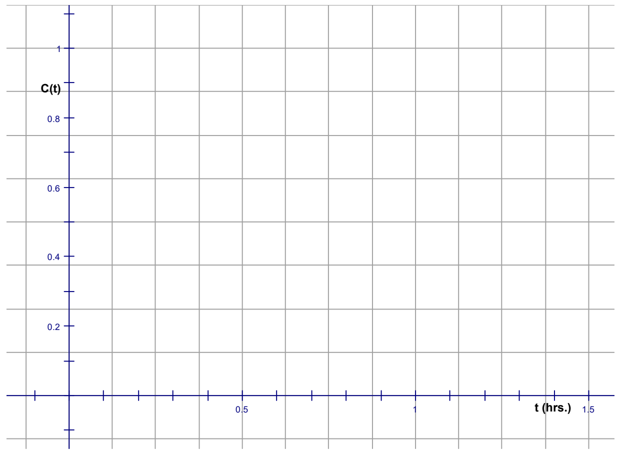
References:
Smith, Minton, Calculus,2nd ed, 2002, McGraw Hill, Sections 2.2, pg. 169, 2.4, pg. 187, 2.6 pgs. 205-208,2.7, pgs. 213-218, and sections 3.3, 3.4, and 3.5.

Taken in part from Nicotine Function: Clinical Pharmacology Online http://www.cponline.gsm.com and from National Institute on Alcohol Abuse and Alcoholism Publication no. 35, PH371 January 1997.

Part I: Consider the graph of the rate of change of nicotine concentration in the blood over time, C'(t), where t is measured in hours since smoking one cigarette and C'(t) is measured in the milligrams per deciliter per hour.

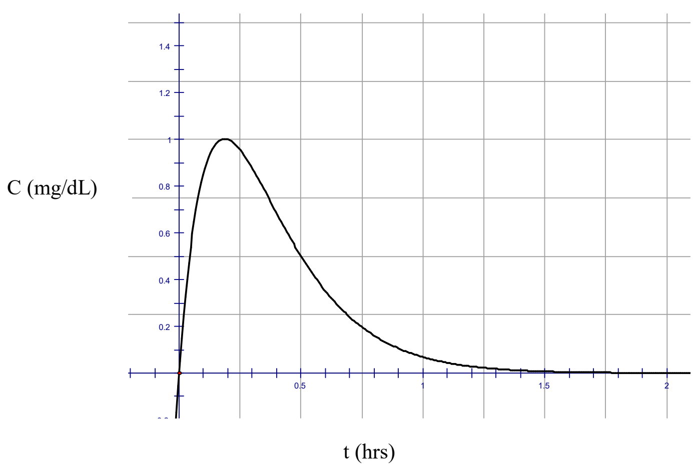


1. On the blank graph below, graph the concentration of nicotine in the blood as a function of time, assuming when t = 0 that C = 0 (at time 0, there is no nicotine in your blood).



2. Give this some thought…is the concavity of your graph correct? How can you tell?

3. The correct graph for C(t) is below. If your graph was not correct, where did you go wrong and why?



4. Based on the graphs of C(t) and C'(t), estimate the following:
    a. Find the time, t, where the maximum concentration occurs.

    b. Find the slope of the tangent line to the curve of C(t) at t=0.

5. Let's give some thought to the function for C(t).
    a. What function would approximate C(t) on the interval [0,0.1]?

    b. What type of function would approximate C(t) on the interval [0.4,2]?

6. The function graphed above is $C(t) = 14.6te^{-5.36t}$. Do the factors in this function correspond to the functions you described in 5a and 5b? Why or why not?

7. At what time is the nicotine leaving your body most rapidly? Answer:

    a. Using the graph of C(t)

    b. Using the graph of C'(t)

    c. Without using a graph (using calculus/algebra)

8. Explain in everyday English (to someone who is not in calculus) what the following mean in terms of the graphs:

    a. $C(t) > 0$

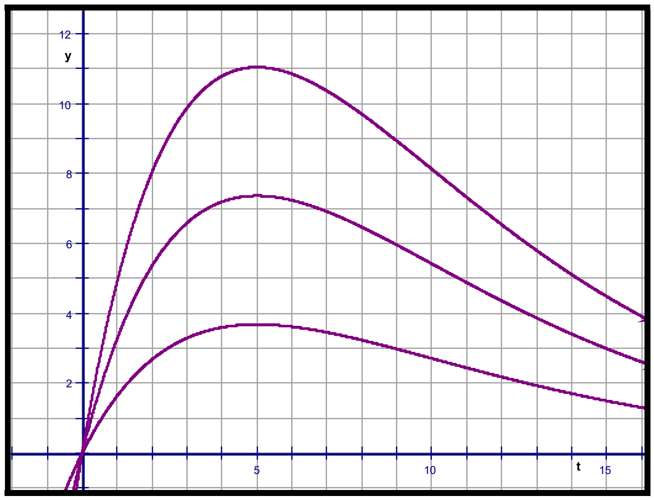
    b. $C'(t) > 0$

    c. $C''(t) > 0$

Part II: The functions of the form y = Ate^{Bt} (like the above nicotine function C(t)) are called surge functions and display an initial approximate linear growth from zero that then (as t increases) become dominated by an exponential decay. Let's consider the general form of these functions, y = Ate^{Bt}.

1. On the graph below label the graphs of each of the following functions:

   $y_1 = 2te^{-0.2t}$ **$y_2 = 4te^{-0.2t}$** **$y_3 = 6te^{-0.2t}$**



2. Briefly discuss what you notice about the shapes of these graphs and about what happens to the graphs as you change the parameter A in the equation y = Ate^{Bt}.

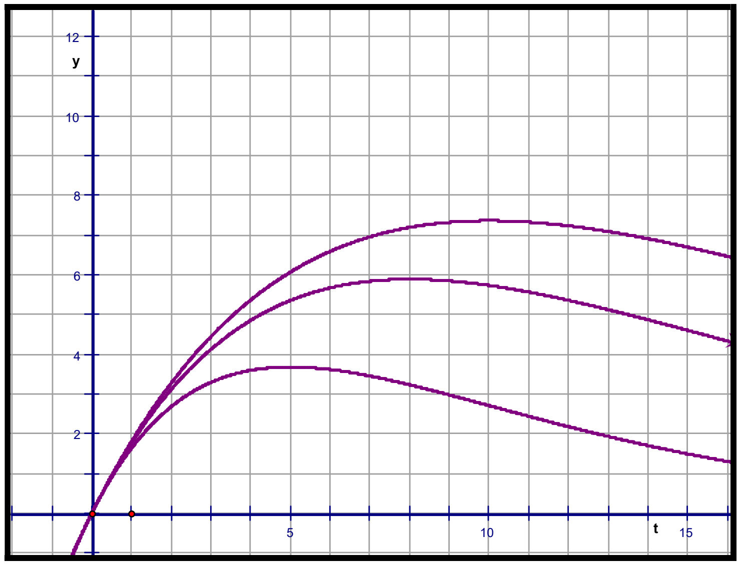
3. For each of the three given functions, use calculus to find the slope of the tangent to the graph at t = 0.

4. For the general equation, y = Ate^{Bt}, what is the slope of the tangent to the curve at 0?

5. Verify your answer in #4 using calculus.

6. On the graph below label the graphs of each of the following functions:

$y_1 = 2te^{-(1/5)t}$ $y_2 = 2te^{-(1/8)t}$ $y_3 = 2te^{-(1/10)t}$



7. Briefly discuss what you notice about the shapes of these graphs and about what happens to the graphs as you change the parameter B in the equation $y = Ate^{Bt}$.

8. For which values of t does it appear that these functions take on their maximum value?

9. For each of the three given functions, use calculus to find the slope of the tangent to the graph at those values of t for which the function is a maximum.

10. Summarize your findings in the table:

Function	Value of t for y_{max}	Slope of tangent line at this point
$y = 2te^{-(1/5)t}$		
$y = 2te^{-(1/8)t}$		
$y = 2te^{-(1/10)t}$		
$y = Ate^{Bt}$		

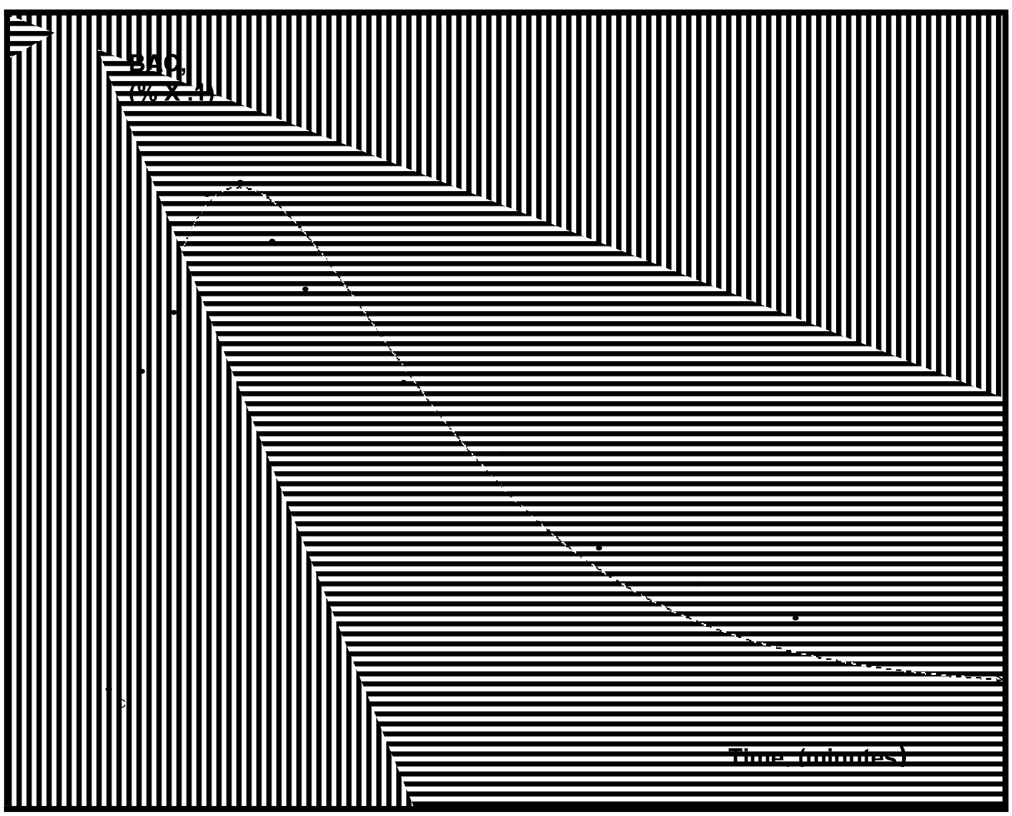
11. Let's revisit the surge function for nicotine levels, $C(t) = 14.6te^{-5.36t}$. Based on what we now know,

- At what time, t, does the maximum concentration occur?
- What is the slope of the tangent line to the curve of C(t) at t=0?
- How do your answers compare with the answers you estimated from the graphs in Part I, #4 (page 2)?

Part III: Now that we understand the A and B parameters in surge functions of the form $y = Ate^{Bt}$, let's examine blood alcohol level data and a possible surge function to model it.

The following data table shows the average of the values of Blood Alcohol Concentration (BAC) recorded over time for a group of male drinkers who rapidly consumed two drinks. For example, a value of 27 at a particular time means that the averages of the values of each participant's BAC at that time was .027%. The given data is plotted below, along with a possible mathematical model of the data in the form of a surge function.

Time (in minutes since the beginning of the drinking period)	0	10	20	30	40	50	60	90	120	150	180	210	240
Blood Alcohol Concentration (2 drinks)	0	27	32	42	43	38	34	26	20	12	9	6	2



1. Using what you have learned about surge functions, write a possible equation that would model this situation.

2. Do you think that this is a good model for the given data? Why or why not?

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