Image of satellite

Calculus 1, Workshop 8: Area Under a Curve – Rescuing A Satellite

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References:
Stewart, Calculus,2nd ed, 2001, Brooks Cole, CA, Sections 5.1.

Warm-up Questions:

1. If a vehicle is traveling at 45 miles per hour in a straight line, what distance will it travel in the first hour?

2. How far from its starting position will the vehicle in question 1 be in:

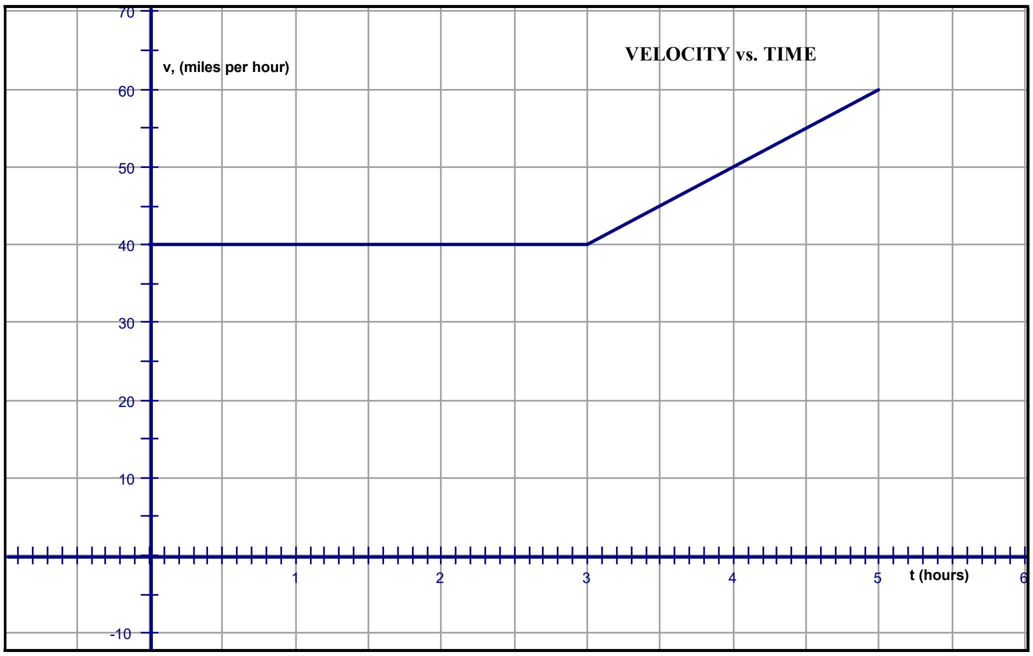
   a. 24 hours?

   b. 3 days (assume 24 hours is one day)?

   c. 2 months (assume one month is $\frac{1}{12}$ year, and 1 year is 365 days)?

d. 24 months?

Consider the velocity vs. time graph shown below for an object traveling on a straight-line path.



3. Is the velocity constant for the first three hours [0,3]; explain?

4. Compute the distance traveled by the object during these three hours [0,3]; how does this relate to the velocity vs. time graph?

5. Is the velocity constant for the next two hours [3,5]? Explain.

6. Compute the distance traveled during these two hours [3,5].

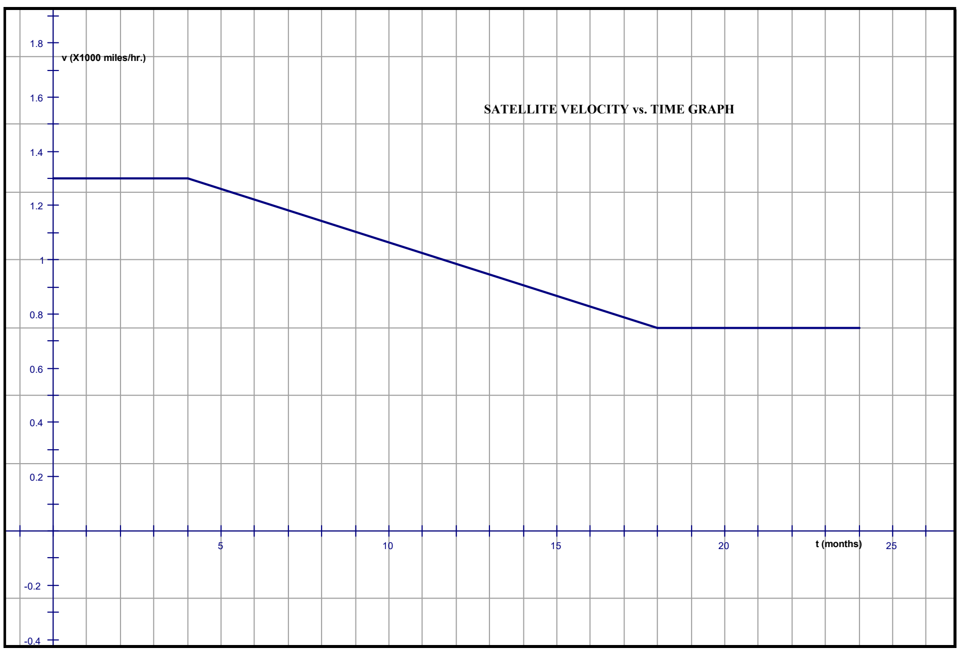
7. Given a velocity vs. time graph for an object traveling in a positive direction in a straight line path, how can you determine its change in position along that path?

The Problem:

An interplanetary satellite that has been sent into deep space has developed severe problems that cannot be solved remotely. If the satellite is not rescued within two years, it will be useless. The satellite is traveling on a straight-line path away from earth, and is currently 100,000 miles away. A rescue ship has been sent, and you need to determine if it will be able to intercept the satellite before it is too late.

Part I: The Satellite in Trouble

The velocity versus time graph for the satellite for the next two years is shown below:



1. Over what time intervals is the satellite's velocity:
   a. constant?

   b. increasing?

   c. decreasing?

2. During these same time intervals state whether the distance from the earth to the satellite is increasing, decreasing or staying the same.

3. How far is the satellite from earth at the end of the first four months?

4. For any time, t, between 0 and 24 months, would you be able to exactly compute the satellite's position?

5. Give the equation of the line that defines the satellite's velocity between 4 and 18 months.

6. The distance traveled by the satellite in any time interval can be determined by computing the area under the v vs. t graph, if we adjust for units. Velocity on the graph is given in units of 1000 mph. The time is in months. We want the area under the graph to have the units of miles (the change in distance over the specified time interval). Complete the following area calculation for distance traveled over the time interval from 0 to 1 month:

$$1300\frac{miles}{hour}\times\frac{24\,hours}{1\,day}\times\frac{365\,days}{1\,year}\times\frac{1\,year}{12\,months}\times 1\,month = \qquad \text{miles}$$

7. Below, the table has been completed using the velocity vs. time graph. Complete the table for the time intervals [12,13]:

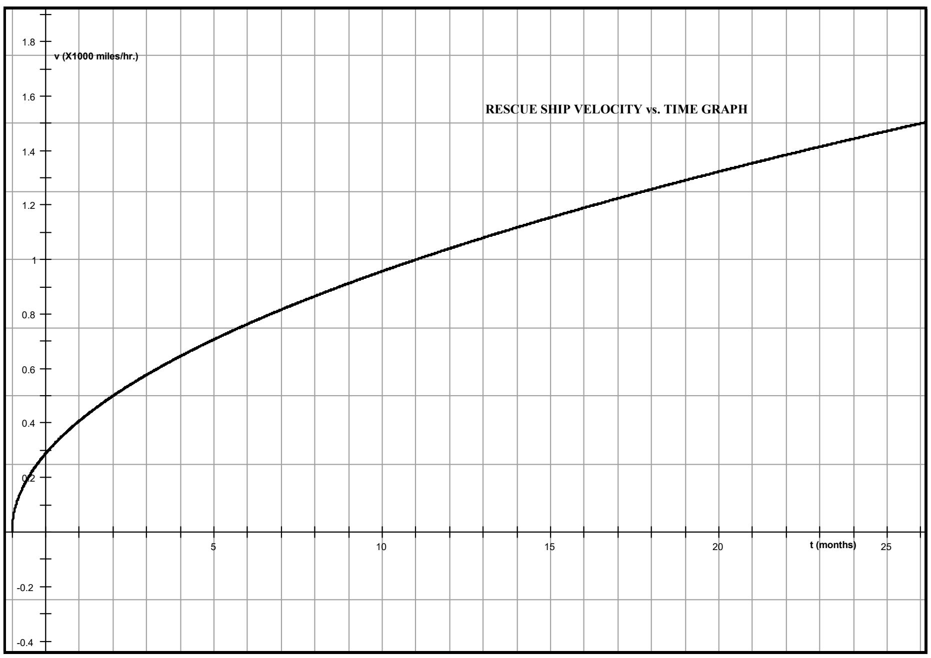
Month	Velocity	Velocity	Distance	Distance from
	Start	End	Travelled	Earth at End
	(mph)	(mph)	(miles)	(miles)
start				100,000.00
0-1	1,300.00	1,300.00	949,000.00	1,049,000.00
1-2	1,300.00	1,300.00	949,000.00	1,998,000.00
2-3	1,300.00	1,300.00	949,000.00	2,947,000.00
3-4	1,300.00	1,300.00	949,000.00	3,896,000.00
4-5	1,300.00	1,260.71	934,660.71	4,830,660.71
5-6	1,260.71	1,221.43	905,982.14	5,736,642.86
6-7	1,221.43	1,182.14	877,303.57	6,613,946.43
7-8	1,182.14	1,142.86	848,625.00	7,462,571.43
8-9	1,142.86	1,103.57	819,946.43	8,282,517.86
9-10	1,103.57	1,064.29	791,267.86	9,073,785.71
10-11	1,064.29	1,025.00	762,589.29	9,836,375.00
11-12	1,025.00	985.71	733,910.71	10,570,285.71
12-13				
13-14	946.43	907.14	676,553.57	11,952,071.43
14-15	907.14	867.86	647,875.00	12,599,946.43
15-16	867.86	828.57	619,196.43	13,219,142.86
16-17	828.57	789.29	590,517.86	13,809,660.71
17-18	789.29	750.00	561,839.29	14,371,500.00
18-19	750.00	750.00	547,500.00	14,919,000.00
19-20	750.00	750.00	547,500.00	15,466,500.00
20-21	750.00	750.00	547,500.00	16,014,000.00
21-22	750.00	750.00	547,500.00	16,561,500.00
22-23	750.00	750.00	547,500.00	17,109,000.00
23-24	750.00	750.00	547,500.00	17,656,500.00

Part II: The Rescue Ship

The rescue ship leaves when the satellite is 100,000 miles from earth. It also follows a straight-line path. The velocity of the rescue ship is defined at any time, t, by the function:

$$v(t)=\sqrt{\frac{t+1}{12}}$$

where t is in months, and v is in thousands of miles per hour. The velocity vs. time graph of the rescue ship is shown below.

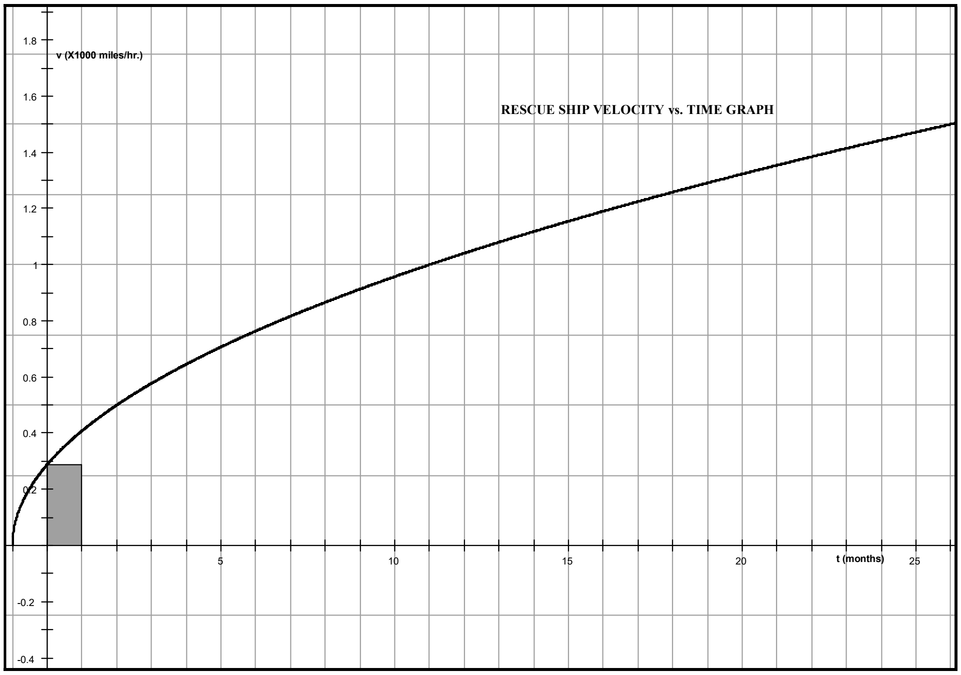


1. Will it be possible to compute the distance traveled by the rescue ship exactly using the velocity vs. time graph, why or why not?

Using the equation, it is possible to compute the velocity of the rescue ship for any time, t. The (t, v) table of ordered pairs is given below for each month [0,24].

t(months)	0	1	2	3	4	5	6	7	8	9	10	11	12
v(X1000mph)	0.29	0.41	0.50	0.58	0.65	0.71	0.76	0.82	0.87	0.91	0.96	1.00	1.04
t(months)	13	14	15	16	17	18	19	20	21	22	23	24	
v(X1000mph)	1.08	1.12	1.15	1.19	1.22	1.26	1.29	1.32	1.35	1.38	1.41	1.44	

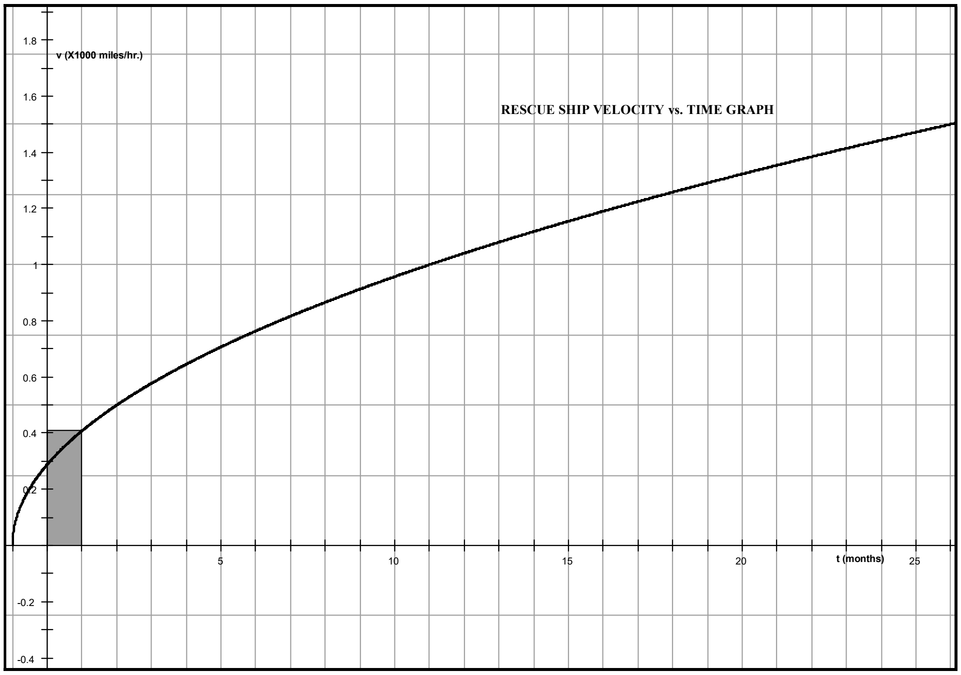
One method of approximating the area under a graph is by drawing a series of rectangles, and computing the area of the rectangles. We will draw 24 rectangles, each with a width of one month. The function value will give us the height of our rectangles. In our first approximation, we will determine the height by using the function value at the left endpoint of the interval. In our second approximation we will use the function value at the right endpoint of the interval.



2. The graph above shows the first of the 24 rectangles needed, when we use the left endpoint as the height. Sketch four or five more rectangles. Will the area of the rectangles be an under or over-estimate of the actual area under the v vs. t graph?

3. The distance traveled during any interval of time is approximately equal to the area of the rectangle used for that time interval. The area of the first 22 rectangles have been used to determine the approximate position of the rescue ship at the end of each of the first 22 months. Complete the table for the last two months. (Recall the conversion factor used for the satellite computations in Part I).

Time Interval	Approx. Velocity (mph)	Distance Travelled (miles)	Distance from Earth at End (miles)
start			0.00
0-1	288.68	210,732.85	210,732.85
1-2	408.25	298,021.25	508,754.10
2-3	500.00	365,000.00	873,754.10
3-4	577.35	421,465.70	1,295,219.80
4-5	645.50	471,212.97	1,766,432.77
5-6	707.11	516,187.95	2,282,620.72
6-7	763.76	557,546.71	2,840,167.43
7-8	816.50	596,042.50	3,436,209.93
8-9	866.03	632,198.54	4,068,408.48
9-10	912.87	666,395.78	4,734,804.26
10-11	957.43	698,921.79	5,433,726.05
11-12	1,000.00	730,000.00	6,163,726.05
12-13	1,040.83	759,808.09	6,923,534.14
13-14	1,080.12	788,490.12	7,712,024.25
14-15	1,118.03	816,164.81	8,528,189.07
15-16	1,154.70	842,931.39	9,371,120.46
16-17	1,190.24	868,873.79	10,239,994.25
17-18	1,224.74	894,063.76	11,134,058.01
18-19	1,258.31	918,563.19	12,052,621.20
19-20	1,290.99	942,425.95	12,995,047.14
20-21	1,322.88	965,699.23	13,960,746.37
21-22	1,354.01	988,424.67	14,949,171.05
22-23			
23-24			



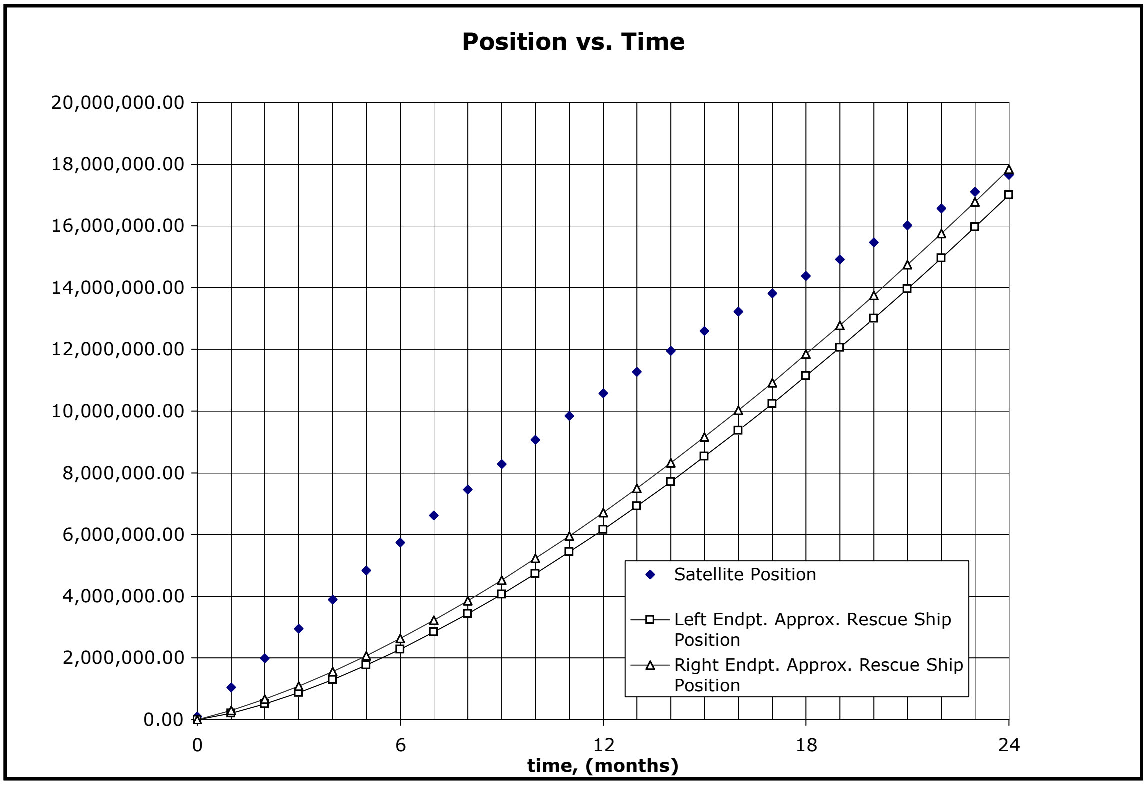
4. Similarly, the graph above shows the first of the 24 approximating rectangles, when we use the right endpoint as the height. Sketch four or five more rectangles. Will a right endpoint approximation give an under or over-estimate of the actual area under the v vs. t graph?

5. The area of the first 22 rectangles has been used to determine the approximate position of the rescue ship at the end of each of the first 22 months. Complete the table for the last two months.

Time Interval	Approx. Velocity (mph)	Distance Travelled (miles)	Distance from Earth at End (miles)
start			0.00
0-1	408.25	298,021.25	298,021.25
1-2	500.00	365,000.00	663,021.25
2-3	577.35	421,465.70	1,084,486.95
3-4	645.50	471,212.97	1,555,699.92
4-5	707.11	516,187.95	2,071,887.87
5-6	763.76	557,546.71	2,629,434.58
6-7	816.50	596,042.50	3,225,477.09
7-8	866.03	632,198.54	3,857,675.63
8-9	912.87	666,395.78	4,524,071.41
9-10	957.43	698,921.79	5,222,993.20
10-11	1,000.00	730,000.00	5,952,993.20
11-12	1,040.83	759,808.09	6,712,801.29
12-13	1,080.12	788,490.12	7,501,291.41
13-14	1,118.03	816,164.81	8,317,456.22
14-15	1,154.70	842,931.39	9,160,387.61
15-16	1,190.24	868,873.79	10,029,261.40
16-17	1,224.74	894,063.76	10,923,325.16
17-18	1,258.31	918,563.19	11,841,888.35
18-19	1,290.99	942,425.95	12,784,314.30
19-20	1,322.88	965,699.23	13,750,013.52
20-21	1,354.01	988,424.67	14,738,438.20
21-22	1,384.44	1,010,639.24	15,749,077.43
22-23			
23-24			

Part III: The Rescue: Yes or No

The position vs. time graphs for the satellite and the rescue ship (right and left endpoint approximations) is shown below.



1. Using this graph, does the rescue ship reach the satellite in time; explain?

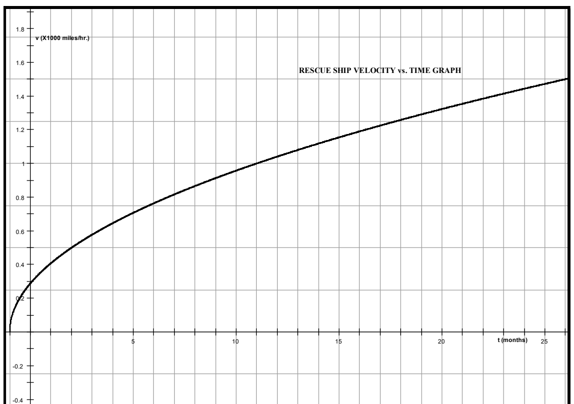
2. If yes, approximate when the rescue takes place; explain your answer.

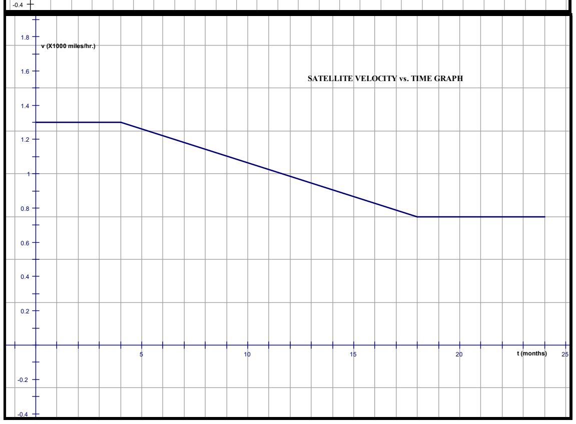
3. How could you refine the approximations that you carried out in this workshop to give a more accurate picture of the position of the rescue ship at any time t?

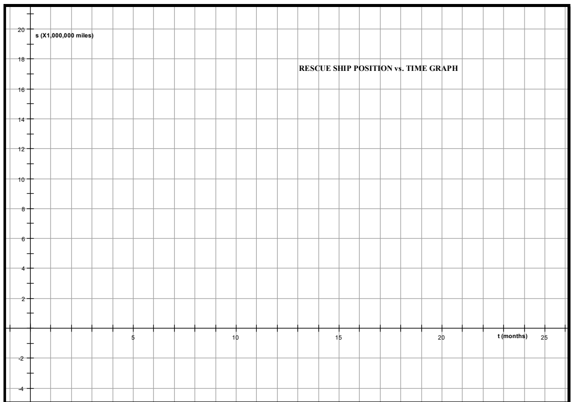
Part IV: Velocity and Position Graphs

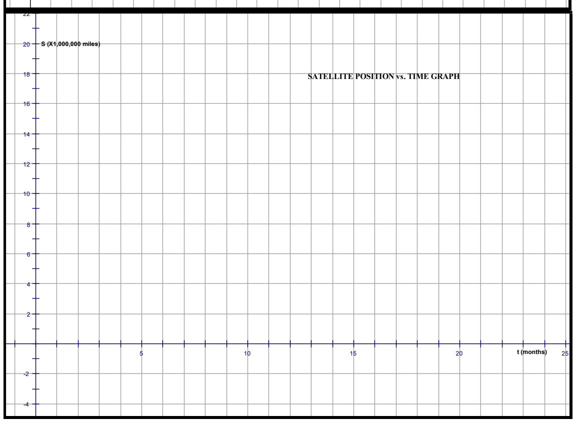
1. Given a position function s(t), how can you obtain an expression for the velocity, v(t)?

2. The velocity vs. time graphs for the satellite and rescue ship are shown below. Using your knowledge of the relationships between a function and its derivative, sketch the corresponding position vs. time graphs for the two. Recall that the initial position of the satellite is 100,000 miles, and the initial position of the rescue ship is 0 miles.









3. How do your position vs. time graphs compare with those shown in part III?

Cite This Module as: Drewniany, P., McGarry, S., Tyne, J. (2012). Peer-Led Team Learning: Calculus I, Workshop 8: Area Under a Curve. Online at http://www.pltlis.org. Originally published in *Progressions: The Peer-Led Team Learning Project Newsletter*, Volume 7, Number 4, Summer 2006.