The Implementation of Vygotsky's Zone of Proximal Development in a PLTL Mathematics Workshop

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The Peer-Led Team Learning (PLTL) Workshop model focuses on engaging students with discussion and problem-solving pertaining to their course work, which strengthens their skills and understanding of the material. Peer Leaders are the vital component of the workshop model, as they are faced with the responsibility of effectively facilitating workshops. Therefore, the success of a workshop is contingent on not only student performance, but also on the performance of the Peer Leader. It behooves the Peer Leader to steer students in the right direction to engage their minds in learning, in thought-provoking discussion, and in problem solving. To do that, the Peer Leader must effectively implement techniques to provide scaffolding for students’ learning. Scaffolding is the application of Lev Vygotsky’s concept of the Zone of Proximal Development (ZPD), which contributes to the success of a Mathematics workshop.

According to Howe (1996), Russian social cognitive theorist and psychologist Lev Vygotsky was born in St. Petersburg, Russia, in 1896. He would come to maturity at the time of the Russian Revolution, and despite the fact that the course of his life was inescapably shaped by the events occurring in the Soviet Union during that time, his work remained firmly rooted in the European school of thought. Vygotsky obtained degrees in Moscow in law and in the humanities. Not long after that, he accepted a position as a teacher in a secondary school, where he became very fond of the new field of psychology. He read avidly and widely on the subject, and came to the attention of the preeminent psychologists of the day.

Vygotsky was soon invited to become a member of the faculty at the University of Moscow, where he became the center of a group that included Luria, Leont’ev, and others who, very like himself, were fervently energized by the prospect of building a new society on socialist principles. The group would eventually lose favor from the Stalinist regime. Vygotsky was a very prolific thinker and produced a body of important work before he died from tuberculosis at the age of 37. Near the end of his short life, Vygotsky wrote Thought and Language, which was published posthumously in Russia. Vygotsky’s work on the formation and development of scientific concepts is mainly found in the fifth and sixth chapters of the book. Vygotsky’s hope was that his study of concept formation and development would lead to further knowledge and understanding of children’s thinking that would, in turn, lead to better methods of teaching (Howe, 1996). Much of Thought and Language serves as a response to the early work of his contemporary, Jean Piaget, more specifically to Piaget’s book Language and Thought of The Child. Vygotsky and Piaget, who were born in the same year, were interested in answering some of the same questions,
although they seek answers through different lenses, Piaget as epistemologist, in quest of the origin of knowledge; Vygotsky, a psychologist, in quest of the origin of consciousness. However, there were many points of agreements between them. They were both constructivists, believing in that each individual constructs his or her own knowledge and meaning (Howe, 1996).

Vygotsky’s Zone of Proximal Development (ZPD) is “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86). It is the gap between a learner’s actual development level as determined by prior knowledge of the task at hand and the ability to independently solve problems, and the level of “potential” development that can be reached through performing tasks and problem solving under the guidance of an instructor or a more capable peer. Within a learner’s actual development level are functions that have already been matured, which is why the learner is capable of performing those functions without guidance. However, beyond the learner’s actual development, and within his/her ZPD, are functions that are not yet mastered but are in the process of maturation. To achieve maturation of those functions, the learner’s ZPD must undergo a shift. As functions mature, more experience and knowledge are acquired, and as a result, the learner is now capable of performing even more complex tasks. By working within the ZPD, a learner is in effect strengthening those “not yet matured” functions – achieving intellectual development. The more a learner works within the ZPD, the more exposure there is to learning and potential development.

Figure 1. The stages of the Zone of Proximal Development theory.
Although the standard definition of Vygotsky’s ZPD noted above helps to explain the core of the construct, it is not sufficient for one to fully appreciate how Vygotsky conceived it. To do that, one must delve into the construct’s historical origins. Vygotsky’s thinking on the ZPD began to crystallize as he confronted issues relating to IQ and IQ testing, which during his time—and not unlike today—was controversial (Dunn & Lantolf, 1998). Educators then assumed, and sadly many in mainstream education continue to assume, that for teaching to be conducted effectively, students had to attain a threshold level of development (Egan, as cited by Dunn & Lantolf, 1998). This threshold level was established by observing students as they performed specific types of tasks (Van der Veer & Valsiner, as cited by Dunn & Lantolf, 1998). Simply, this approach takes into account only the learner’s actual developmental level and ignores anything that lies at the actual development’s upper boundary. This view of how students achieve intellectual development is of course contradictory to the ZPD construct. Vygotsky, in Language and Thought, argued that one could not fully comprehend a learner’s developmental level without taking into account that development’s upper boundary, beyond which lie tasks that a learner cannot perform without guidance. Taking into account what lies beyond this boundary is important. For example, two students might have identical IQ scores, supposedly indicating that they have attained an actual developmental level, but it is quite possible that one of them might be able to perform more complex tasks under the guidance of an instructor or a capable peer than the other student can with the same level of guidance. This difference between actual and potential IQ is what essentially the ZPD is. Vygotsky’s major insight regarding the ZPD was that instruction and learning do not ride on the tail of development but instead blaze the trail for development to follow (Dunn & Lantolf, 1998).

The construct of scaffolding is closely related to Vygotsky’s ZPD. In fact, the two are often mistakenly referenced interchangeably, although they are based on similar but essentially different elements. The term “scaffolding” was first used by Vygotsky and Luria in reference to how adults introduce children to cultural means (Van der Veer & Valsiner, as cited by De Guerrero and Villamil, 2000) and was later popularized by Jerome Bruner, who was influenced by Vygotsky’s work, and shared his belief that a learner’s social environment and social interactions are key components in the process of learning. As scaffolding is important to the construction of a new building or structure, so is it to the process of learning. The construct emphasizes the importance of proving learners with adequate support in the process of learning new concepts. Scaffolding is very dynamic and responsive. It is imperative that the “right” amount of support is provided at every point of the learning process. Excessive or insufficient support is not ideal – the right balance must be kept at all times. To visualize why providing the right amount of scaffolding is important, it helps to combine the concept of ZPD and scaffolding in a synergic manner. To illustrate: At the upper level of the ZPD lie tasks that a learner cannot achieve without adequate scaffolding or help. Insufficient scaffolding at that level is unproductive because the learner is still not able to achieve those tasks. On the other hand, excessive support is also not productive as the learner is not being challenged. In fact, excessively applied scaffolding can even be futile, as it deviates from the most fundamental purpose of the construct – a way to work within the ZPD.

Construction of knowledge: The ZPD is systemic and multi-relational at its core. It is a tripartite construct – encompassing social, historical, and cultural dimensions that are intricately connected. Each dimension plays a crucial role in development and acquisition of new knowledge (new system formation) (Levykh, 2008). To fully appreciate the complexity of the ZDP, one must understand Vygotsky’s thinking
of culture development that catalyzes the formation of higher mental functions. According to Vygotsky, the construction of knowledge is a continuous process that originates from the past (prior knowledge), to the present, and into the future – hence the historical dimension of the ZPD construct. Such a process is not a continuum, where one stage or level of development is not perceptibly different from another stage or level of development. It is rather a more indirect pattern that emerges as one struggles to form meaning of the world.

“Students create their own knowledge and develop mathematical meanings as they learn to explain and justify their thinking to others. As they learn to speak the mathematical language, they transform their thinking of the mathematical concept”(Steele, 2001, p. 404). In other words, higher mental functions do not instantly emerge from lower mental functions as one interacts with the environment, but rather, that interaction with the environment only initiates the development of those higher mental functions that take time to fully develop. Every new system formation emerges from a dissonance, which can stem from any dimension of the construct, as one navigates the world and “struggles” to form meaning during a period of uncertainty. Imprinted with the newly formed system is its psychological origin, and at the core of which is its meaning.

One cardinal rule of the PLTL model is that Peer Leaders do not directly provide students with answers. If called upon to aid in solving a problem, the Peer Leader should guide the student to obtain the answer, rather than simply providing it. In the context of new system formation, one can see why this approach is conducive to learning. By guiding the student through his or her struggle to obtain an answer, the Peer Leader is helping the student to learn and form his or her own meaning of the challenging concepts at hand. As higher mental functions become embedded within an existing system, they can both overlay on, or reorganize the more basic mental functions. “According to Vygotsky, since every newly formed system is situated within the relational development of other systems, its features come from and are infused in all other systems and subsequently appear in the new systems as well” (Levykh, 2008, p.88). Such a dynamic, multi-relational, systemic, and ever-evolving model of development is not only influenced by the formation of new systems, but also in the ever-evolving intricate connection between old and new systems. This systemic plasticity is responsible for the inter- and intra-dependence of dynamic reciprocity that exists between the emergence of newly formed systems and the further development of its parts. As new systems of meaning evolve, their unique meaning is very important, as mental internalization through personal experience as one navigates the world, then becomes the basis of what creates an individualized personality.

Mutual scaffolding: The traditional ZPD construct assumes the novice-expert scenario, where the novice (learner) is helped by the expert (that is, a more capable peer, or instructor) to solve problems. However, this original model can be altered by using alternative approaches to problem solving. The traditional ZPD novice-expert scenario can be altered to a novice-novice scenario, where two novices are now brought together to facilitate the learning and problem solving of one another. “Whereas studies on scaffolding have traditionally examined tutor-learner situations, Donato (1994) explored the notion of ‘mutual scaffolding’ among 2nd Language (L2) learners” (De Guerrero and Villamil, 2000, p. 54). Donato’s purpose was to observe to what extent three novice students of French, working collaboratively on a task, could positively influence each other’s development in the foreign language.” As mentioned, scaffolding is a way of working within the ZPD, and in the mutual scaffolding approach, it is bi-
directional rather than unidirectional. The two learners serve both roles of novice and expert, reciprocating scaffolded assistance as they are both working within their respective ZPDs. This approach is optimally effective when the tasks at hand do not lie too far out of the actual development level of either novice, where their combined competence would not suffice to complete such tasks. In such a case, the assistance of a third party (for example, a Peer Leader or instructor) would be required to achieve completion of the tasks. In the ideal case of mutual scaffolding, two novices with similar ZPD work cooperatively, providing scaffolded assistance to one another reciprocally. As they progress to complete the task at hand, they consistently support and scaffold each other’s learning.

In a study at the Inter American University of Puerto Rico, the traditional novice-expert ZPD scenario was altered by bringing together two novice ESL students working jointly to revise a text written by one of them. Strategies of revision took shape and developed in the inter-psychological space created when two learners are working within their respective ZPDs. The study concluded that in second language peer revision, scaffolding may be mutual rather than unidirectional (De Guerrero and Villamil, 2000).

**Observations**

In the first credit-bearing mathematics course, covering algebra and geometry (MAT 1175), several strategies were used.

**Peer Pairing**

In the first weeks of workshops, students were solving problems at noticeably different paces. Some were consistently breezing through the material while others had difficulties keeping up. As the Peer Leader, I paired slower-paced students with more capable peers and instructed the pair to discuss precisely the approach best suited to solve the problems. As a result, slower-paced students were able to learn the material with the help of more capable peers with slightly higher ZPDs.

**Scaffolding by Hints**

**Scenario:** During a workshop session, a student trying to solve the problem below stalled and was uncertain what steps would lead to the solution.
Problem 1: In the figure above, angle A = angle C. \( AB = x^2 - 15 \), \( BC = 2x \).

a) Write the congruence statement and give a reason (SAS, ASA, AAS or SSS Theorems).

b) Find \( x \), \( AB \) and \( BC \). (Ghezzi et al., 2012)

Student: I am not sure how to solve this problem [referring to part b) of the problem].

Peer Leader: Well, why don’t you write down the “givens” of the problem?

[Student writes down the “givens” of the problems, marking angles A and C as equal; and labeling sides \( AB \) and \( BC \) in terms of \( x \).]

Peer Leader: So angle A and angle B, the base angles of the triangle, are equal. From that fact, what can you conclude about this triangle?

Student: The triangle is isosceles.

Peer Leader: Yes! What more can you say about the triangle?

Student: Sides \( AB \) and \( BC \) are equal.

Peer Leader: Correct!

[Student is able to see the relationship existing between sides \( AB \) and \( BC \) and sets the equation \( x^2 - 15 = 2x \) (Equation 1).]

From Equation (1), the student is then able to solve the problem and obtain the values for \( x \), \( AB \), and \( BC \).

In the example above, the key to solving the problem is identifying the triangle as isosceles. As soon as the student was able to do that, each subsequent step followed smoothly. All it took on my part was just a few helpful hints.

(Calculus I (MAT 1475): Scaffolding within the ZPD)

Scenario: A student solving the problem below successfully solved the first part (computing \( f(g(x)) \)) and did not know how to approach the second part of the problem (finding out the domain of the composite function computed in part a)), although he had a clue.

Problem 2: If \( f(x) = \sqrt{x - 2} \) and \( g(x) = x^2 + 6 \)

a) Find \( f(g(x)) \)

b) Find the domain of \( f(g(x)) \)

Student: I am not sure how to find the domain of the composite function.

Peer Leader: Ok, the composite function is \( f(g()) \). I can tell that your answer to part a) is correct, can you explain how you derived it?

Student: I started by plugging in \( g(x) \) inside of \( f(x) \).

Peer Leader: Ok, that is the correct procedure, but since we are plugging in \( g(x) \) inside of \( f(x) \), what do we first have to know about \( g(x) \)?

Student: The domain of \( g(x) \)?
Peer Leader: Yes, since we are plugging \( g(x) \) inside of \( f(x) \), we first have to know if \( g(x) \) is undefined for any values of \( x \) because we would not be able to calculate \( f(g(x)) \) at that value. In other words, we cannot calculate \( f(g(x)) \) for a value of \( x \) where \( g(x) \) is undefined.

[Student successfully calculates the domain of \( g(x) \); domain \( g(x) = x^2 + 6 : -\infty < x < \infty \) ]

Peer Leader: Ok, you have successfully commuted the composite function and now know that \( g(x) \) is defined for all values of \( x \), what is the next step here?

Student: Calculating the domain of the composite function?

Peer Leader: Yes!

[Student successfully calculates the domain of the composite function \( f(g(x)) \); domain \( f(g(x)) = \sqrt{x+6} - 2 : -\infty < x < \infty \) ]

Peer Leader: So what is the domain of \( f(g(x)) \)?

Student: All real numbers.

Peer Leader: Correct!

Peer Leader: So what is the general procedure for computing the domain of a composite function in the form \( f(g(x)) \)?

Student: Computing the domain of \( g(x) \) and then the domain of the composite function \( f(g(x)) \).

Peer Leader: Yes, because the domain of the composite function \( f(g(x)) \) will not be defined for any values restricted by the domain of \( g(x) \) along with any values restricted by the composite function \( f(g(x)) \) itself.

In the example above, the domain of \( g(x) \), the starting function, is equal to the domain of \( f(g(x)) \), the composite function. In this case, the student could have erroneously concluded (or guessed) that the domain of \( f(g(x)) \) is obtained solely from calculating the domain of \( f(g(x)) \) itself, without taking into account the domain of \( g(x) \), and would still correctly answer the question. However, it is not always the case where the domain of the starting function is the same as the domain of the composite function. For that reason, it is important that the student fully understands the process of computing the domain of a composite function.

Discussion

A learner’s ZPD bounds the upper limit of a student’s actual development zone, and beyond that boundary lie tasks a student cannot successfully complete without help. When the problems or tasks shift a learner out of his or her actual development zone and into his/her ZPD, different techniques can then be employed to facilitate the student’s learning. I have found through facilitating workshops that not all students learn alike. For example, some prefer graphic explanations while others may prefer model-building. For that reason, no one technique is a panacea for working within the ZPD. However, deviating slightly from the direct approach of a particular technique sometimes suffices to accommodate the needs of students who approach problems in various ways. Below are some of the techniques that have proven to work in my workshop sessions. It is important to keep in mind that no one technique is ideal in every situation.

Peer Pairing, as the term implies, involves having two students work cooperatively to solve problems. For this technique, two roles are assigned (or at least assumed): more-capable peer, and slower-paced
peer. The more capable peer is more knowledgeable about the topic at hand and has the ability to help the slower-paced student. In the ideal case, the more-capable peer is adequately knowledgeable and skilled at the tasks at hand that he/she is never working within his/her own ZPD. Otherwise, the intervention of a third party (for example, the Peer Leader) would be required. Although it is best that the more-capable student does not encounter problems or tasks so challenging that he/she would have to work within his/her own ZPD, the problems or tasks should challenging “enough.” The more-capable peer should be working within the upper landscape of his/her actual development zone. That is very important, because if tasks/problems are “too easy,” then the more-capable peer is then enacting the role of the peer leader more than the actual role of being the more-capable peer. This technique is beneficial to both student: the more-capable peer strengthens his/her knowledge of the material by problem solving, justification, and explanation as he/she scaffolds the learning of the slower-paced student; and the slower-paced student is able to solve problems and learn material through the help provided by the more-capable peer.

*Scaffolding by hints:* One of the cardinal rules of the PLTL model requires that a Peer Leader never give a direct answer of a problem to a student, rather, it behooves the Peer Leader to “guide” the student to the appropriate answer. It is crucial that the Peer Leader adheres to this rule, for very important reasons. One of those reasons pertains to the very fundamental aspects of how we learn, which is by forming our very own meaning to concepts and ideas. This is why it is also possible to learn concepts the “wrong way” – by wrongly forming contextual meaning. From that notion, we can see why directly providing an answer to a student is not helpful to learning. However, a more clever approach is to guide the student to obtain the right answer by gradually hinting at it. This is much more conducive to learning than just simply providing answers. It is important to mention that the Peer Leader is not a hinting machine. The process is conducted gently and gradually as the Peer Leader accesses the student’s prior knowledge, poses appropriate questions, and helps the student to form contextual meaning of the material at hand.

*Mutual scaffolding* is done similarly to peer pairing, except that the help provided is bi-directional and the two students have similar ZPD’s regarding the task at hand. This approach is best employed in situations when the task at hand is moderately challenging (residing at the upper area of the students’ actual development zones), but not too challenging that they lie beyond either one of their actual development zones and into the ZPD. The closer the tasks are to the upper limit of the students’ actual development zone, the more cooperation and interaction is required on their part to progress and achieve completion of a task. For tasks that are too easy, not much interaction or cooperation is required as the two students would be capable of easily completing the task individually. That is why it is important that the tasks at hand be challenging and require effort and cooperation from both students. As they progress toward completion of a moderately challenging task, they provide help to one another reciprocally by supporting and scaffolding each other’s learning. This approach encourages student engagement and interaction, and is the underpinning to the PLTL model; both students benefit tremendously.

*Assessment of prior knowledge* plays a crucial role in the acquisition of new knowledge. In learning, it is what helps to “connect the dots” to form contextual meaning of new or unfamiliar concepts. From that notion, it is clear to see why accessing a student’s prior knowledge as he/she progresses to complete a task is very helpful and conducive to learning. By carefully posing appropriate questions, the Peer Leader
is able to help a student to recall prior knowledge that can be expanded to help complete the task at hand. When done properly, this approach subtly hints to the student how the prior knowledge is related to the current task and how it can be used to complete that task. From that point, the student is able to connect the dots and progress to complete the task. This approach is doubly effective when used in conjunction with scaffolding by hints.

Conclusions
Through facilitating workshops, I have become not only a better leader but a better learner as well. I did not foresee learning as much as I did from my students. The satisfaction of helping others succeed is an indescribable feeling and I look forward to doing it all over again. Can’t wait!!!

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