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HOW PEER-LED WORKSHOPS CAN DEVELOP STUDENTS' MATHEMATICAL CONCEPTIONS BY UNDERSTANDING THEIR MATHEMATICAL MISCONCEPTIONS

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Some knowledge of mathematics is required for all college students. Sadly, about half of the students taking the first credit-bearing mathematics course (Fundamental of Mathematics, MAT 1175) at New York City College of Technology ("City Tech"), City University of New York (CUNY), do not pass the course the first time they enroll. What causes this failure? As a first-time Peer Leader in the Spring 2014 semester, I conducted a qualitative survey in a MAT 1175 class that included PLTL workshops. Nineteen (19) students responded, and the survey results showed that students use class notes, formulas given in class, and other types of assistance to build their mathematics concepts. Unfortunately, the misconceptions that students already have interfere with new learning which could be a possible reason for their failure. A second study in the Fall 2014 semester was conducted to see the source of mathematical misconceptions along with the effect on learners. Professor Sandie Han, who taught the class, provided one set of students' quizzes and one set of test papers to be analyzed. One problem from each set, solving radical equations, was chosen for the second study. Results suggest that undeveloped conceptual learning as well as misunderstanding rules and formulas may be sources of misconceptions which cause lower grades for the students.

Individuals construct their knowledge base using older constructs in new situations (Derry, 2013). Individuals' active participation in problem solving and critical thinking are necessary for learning, which allows adaptation: what is already understood connects to newly learned information (Eison, 2010). Lev Semanovich Vygotsky (1896-1934), Russian psychologist, focused his research on intellectual development and examined how learning is supported by social interaction. Vygotsky explained the learning of new knowledge through the Zone of Proximal Development(ZPD). The lower end of the ZPD refers to the concepts that a student can learn without any assistance. The higher end of the ZPD contains concepts that a student cannot learn, even with assistance. The goal of a learner is to move from the bottom of the ZPD towards the top through any type of assistance. Children are able to perform a series of actions (which surpass their own capacities, but only within limits) better when they work together and are guided by others rather than when alone (Daniels, 1996).The support whereby a learner takes external information that is being learned and internalizes it, integrating it with one's own knowledge is supported by *scaffolding*– any type of assistance.

Learning is affected by the learner's environment and depends upon prior knowledge. Moreover, the development of mathematical concepts in a learner depends on achieving a particular level of maturation of *spontaneous concepts* (Zack, 1999); these are knowledge learned without realization because they happen in the subconscious mind. For example, learning a mother-tongue for a child is a spontaneous way of learning a language. Mathematics is a language with its own symbols, and some modalities of learning mathematics are spontaneous (Hasan, 2001). More advanced mathematics concepts inevitably influence existing spontaneous concepts (Zack, 1999). For example, learning calculus concepts may influence the spontaneous concepts of calculating algebra or graphing functions.

However, misconceptions originate from miscomprehended prior knowledge or misunderstood spontaneous concepts, and can be stable (Smith, diSessa, & Roschelle, 1993-1994). These misconceptions can be widespread among students and interfere with new learning. Consequently, students' flawed ideas conflict with the basic premise of constructivism. For example, we know a basic rule for math that only like terms can be combined, or humorously, we cannot combine apples and oranges. Yet, what if someone were to ask to combine apple and apple juice – are they same? In mathematics, we often see the algebraic expressions where a constant is combined with a variable such as $16 + 16x$. What should we do here? Can we combine both "16s"and the answer will be $32x$, or not? Misconceptions create confusion for students in trying to solve a problem.

Students' misconceptions can be generated naturally from the lack of basic Mathematics concepts. An empirical study by Koedinger, Alibali and Nathan(2008)found that students are more successful on story problems by using informal and traditional strategies to unwind the problem and they are less successful learning symbolic equations. The authors categorized the story problem as *grounded representation* and symbolic problems as *abstract representation*. Higher levels of mathematics are mostly abstract representations of mathematics, and such representation does not have physical referents in which to ground a concept, creating misconceptions. Koedinger et al. (2008) showed that students have difficulties with syntax, semantics of equations, and comprehending and manipulating algebraic expressions in higher level courses (2008). Similarly, students may treat numerical variables as objects instead of numbers, another source of potential misconceptions (Swift, 2013).

Egodawatte (2011) conducted a mixed method study of high school students and found that students not only have a lack of understanding of basic math concepts, they also have problems in understanding accepted rules or algorithms for abstract structures of algebraic equations. Inadequate understanding of the uses of the equal sign and its properties led to major errors in solving equations. Egodawatte suggested that students need the correct conceptual knowledge (why), procedural knowledge (how), conditional knowledge (when), based on the stage of the problem solving process, which will allow students to avoid errors and misconceptions. Conceptual knowledge will answer why students would follow an algorithm for a particular problem, procedural knowledge will tell them how to apply the algorithm, and conditional knowledge will let them know when to use the algorithm.

Methodology

Over two semesters, two studies were conducted. The first, in Spring 2014, was a qualitative questionnaire exploring the methods used by students in solving mathematics problems. The second, in the Fall 2014 semester, examined students' written work on a problem of solving a radical equation, both on a quiz and on an exam.

First Study

The first study was conducted on April 10, 2014 in Professor Sandie Han's Spring 2014 mathematics class (Fundamental of Mathematics, MAT1175, covering algebra concepts and geometry) in her section with a one-hour PLTL workshop component. Nine questions were asked inquiring about the development of mathematical concepts. Nineteen (N=19) students completed the open-ended questions.

Results from the questionnaire

The responses to the open-ended questions were categorized as shown below in Tables 1-9.

Table 1. Question 1. What is your major?

[Responses were grouped according to the three Schools at City Tech]

School	Responses (n=16)
School of Arts and Science	4 (Biomedical Informatics[BT], Liberal Arts &Sciences [AA], Mathematics[BT])
School of Professional Studies	7 (Marketing[BA], Health, Radiology Technology)
School of Technology and Design	5 (Electrical Engineering Technology, Electro-Mechanical Technology [AS], Architectural Technology)

*[**Associate level Degree = AA, Bachelor level Degree = BT, BA]*

The respondents, students in the first credit-bearing mathematics class, are evenly distributed from each of the three schools, and their majors represent a mix of Associate and Bachelor degree programs.

Table 2. Question 2. What Mathematics courses are required for your Major?

Course	Responses(n=49)
Fundamentals of Mathematics (MAT1175)	19
Intermediate Algebra and Trigonometry (MAT 1275)	13
Pre-calculus (MAT 1375)	9
Calculus I (MAT 1475)	5
Calculus II (MAT 1575)	3

Depending on their majors, all the respondents need to pass this course to continue with the math sequence, even if most are only required to take one more course, and nearly half must complete Pre-Calculus; one quarter need to complete Calculus I.

Table 3. Question 3. Is math important to you? Why or why not?

Categories		Responses (n=23)
YES		14
	Helps to predict and calculate things/needed in everyday life/ crucial to technology	6
	Required to major/factor in education	6
	Required to major/factor in education	2
	Develop skills/ want to master it	3
NO		6
	Too complicated and specific/ difficult	2
	Abstract calculations seems pointless	1
	No, don't enjoy learning/ don't like it	2
	Reconsider my major for certain difficulties	1

The majority of the responses (14 of 43) provided reasons that they understood mathematics was important to their majors, while six did not think it important.

Table 4. Question 4. What do you do to understand a math problem?

Methods	Responses (n=33)
Identify/ look at the information given/ look at it multiple time/ identify missing information	6
See what I can use to solve the problem/ what formula fits/ read the problem	4
Do similar problem/ practice	3
Follow the steps/ learn/ study/ review	8
Use own logic to figure out possible solution	4
Look at class-notes/ look over home-work problems/ draw diagram	4
Try/work on paper	3
Ask for help	1
N/A	2

Respondents provided several methods they use to understand a problem; many of the strategies suggest imitation of an example that has been provided.

Table 5. Question 5. What type of problem can you finish comfortably without struggling too hard?

Problems	Responses (n=22)
All that has been covered so far in MAT 1175/anything I have mastered	2
Liner equation/ quadratic equation and formula/ binomial/factoring	8
Radicals	2
Beginning of algebra/ algebra	4
Don't understand/ N/A	2
Geometry (easy)	1
Depends	1
Easily understandable problems/ anything that's not abstract	2

Respondents answered both conceptually and procedurally: specific types of problems were named, as well as categories of problems (e.g., "easily understandable problems").

Table 6. Question 6. What type of problem have you started to work on, but could not finish?

Problems	Responses (n=20)
Geometry/ finding angles/ congruence of triangles	9
Misunderstanding problems/ proofs	5
Factoring/factoring type equation	2
Long polynomial	1
Radicals	1
Elimination method	1
Lots of problems in small time limits	1
Don't remember/ N/A	3

Respondents stated some conceptual and analytical problems that seem doable in the beginning but unfinishable later. Students also mentioned some reasons for not finishing the problems, such as misunderstanding problems, and too many problems to be solved within short time limits,.

Table 7. Question 7. What type of assistance might you want to finish the problem?

Assistance	Responses (n=25)
Professor/peer leader	12
Workshop/ tutoring/ study group	5
Understanding concepts	2
Review of the problem	1
More time	1
Assistance for checking answer	1
None	3

Respondents mentioned the assistance they want, and responses suggest that this is obtained through help from the professor, the peer leader, and the peers in workshop and informal study groups.

Table 8. Question 8. What type of problem stumps you so you don't even know how to start working on it?

Problems	Responses (n=21)
None/ Rarely happens	8
Radicals/equations/sign number	3
Congruence of triangles/Geometry/angles/area of triangles	6
Unpleasant past/ experience	1
Topics of study	1
Test questions	1
A problem that I don't understand	1

For those respondents who wrote that they were stumped, ten mentioned some conceptual and analytical types of problems, and one responded with the association of unpleasant past experience with problems that seemed incomprehensible.

Table 9. Question 9. Do you think after understanding a problem you can do another similar one by yourself? When have you tried this?

Internalize Areas	Responses (n=25)
Yes	**15**
After teacher teaches / practice in class	5
After learning something new	3
Practice at home/ home-work	5
Difficult to do in class	1
Attempt in workshop	2
Quizzes/ exam/ tried before an exam	3
To make sure	2
To find similarities/after one next should be easy	2
No/ NA	3
Forget some steps/get stuck with certain steps	1
Afraid of minor errors/afraid to try a new problem/ afraid to be mistaken	1

Most of the responses (15 out of 19) thought they can solve problems after doing a similar one. When they had tried to do this included after the instructor had taught the class, after learning something new, while doing homework, at workshop, during test, and self-studying or group studying time. Four of the responses admit to forgetting steps, getting stuck, and being afraid: of minor errors, to try a new problem, to be mistaken.

Second Study

In the Fall 2014 semester, two sets of students' work from the same class (Spring 2014), taught by Professor Sandie Han, were used to observe the students' mathematical misconceptions. The students' work consisted of a quiz (#4), and an exam (#3, or the test before the final). The names of the students were masked by a third party prior to analysis. A similar problem on solving radical equations was selected from each set. There were 23 quizzes and 22 exams from which the problem was used.

Quiz problem:

Solve for x and check: $\sqrt{2x+1}+7=x$

Test Problem:

Solve the radical equation and showcheck: $\sqrt{x+5}+x=7$

On both the quiz and test, students were asked to rate their confidence level, and to rate their expected performance, both on a scale of 1-5.

Findings on Quiz Problem and Test Problem

In both problems, students who solved the problem correctly followed the seven-step algorithm below, shown below in Table 10.

Fourteen students (61%) followed seven steps correctly in the quiz problem. In addition, nine students (39%) made errors in one or more specific steps. On the other hand, thirteen students (59%)followed seven steps correctly in the test problem. Unfortunately, nine students (41%)made errors and they mistook in different steps in the test. Similar types of misconceptions were found in both problems.

Table 10 represents only the students who made errors in each step. It is important to remember that although these students made errors, it does not mean that they made errors in each step. Some steps were correct and their answer was partially correct.

Table 10. Seven Steps to Solving a Radical Equation Problem

No.	Step	Number of students with incorrect responses in the Quiz (n=9)	Number of students with incorrect responses in the Test(n=9)	Comments
1.	**Make a well-balanced equation so the radical functions only stay in one side of the equal sign**	Eight of the nine students did this step correctly; one did not.	Eight students of nine also made similar mistakes in this step.	The student who made the error did not have the concepts to start the problem correctly.
2.	**Take the square on both side of the equal sign so the radical function can be solved by using general mathematical formulas**	Six students misunderstood or jumbled rules.	Four students showed misconceptions in this step in the test	This is a new concept for the students which caused misconceptions, such as $(x-7)^2 = x^2 - 7^2$, $(\sqrt{2x+1})^2 = 4x+1$, $(\sqrt{2x+1}+7)^2 = 2x+1+7$ and $(7+x)^2 = \pm(7+x)\,(7+x)$.
3.	**Apply the distribution property if applicable**	Four students had errors in multiplication and addition, One student could not get the step for making error in the previous steps, One student made a careless error	Three students made mistakes	Third step was most important. In the quiz, the error of distribution of negative sign was found, such as - (a+b) = -a+b.
4.	**Make a quadratic equation**	Two out of nine students did not get the fourth step,	One student did not have the step at all,	Some students missed the step so any analyze could not be made.

No.	Step	Number of students with incorrect responses in the Quiz (n=9)	Number of students with incorrect responses in the Test(n=9)	Comments
		Three students did not get the required equation because of the errors in previous steps	Two students made error	
5.	**Factor or use quadratic formula to solve**	Five out of nine students didn't get fifth step because of the errors in previous steps.	Three out of nine students didn't get the step.	Students who did the previous step incorrectly, they had basic idea about this step which is factoring.
6.	**Solve the equation**	Two students had misconceptions about solving quadratic equation. One of them knew the formula but couldn't apply it properly.	Three students made errors in this step.	Even though nine of them get the wrong final answer, seven of them in the quiz knew the process to solve an equation. On the other hand, six students from the test understood the idea of solving an equation.
7.	**Check the answers (roots) are correct or not**	Two students had misconceptions about checking.	Two students made careless mistake in checking.	Students seemed to give up at this step when they found the answer they got is wrong.
		Three students gave up doing the problem, so didn't do this step	Two made error. One of them didn't complete the checking process and other copied the equation improperly.	

Analysis of Problem-Solving

Students did not have misconceptions in every step of the algorithm. The results are displayed in Figures 1 and 2.

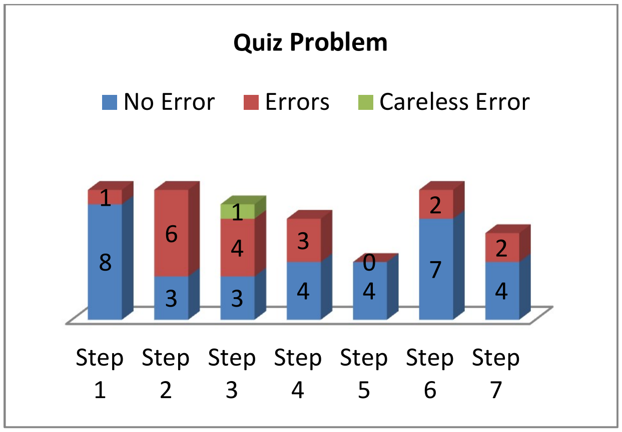

Figure 1: Errors in Quiz Problem

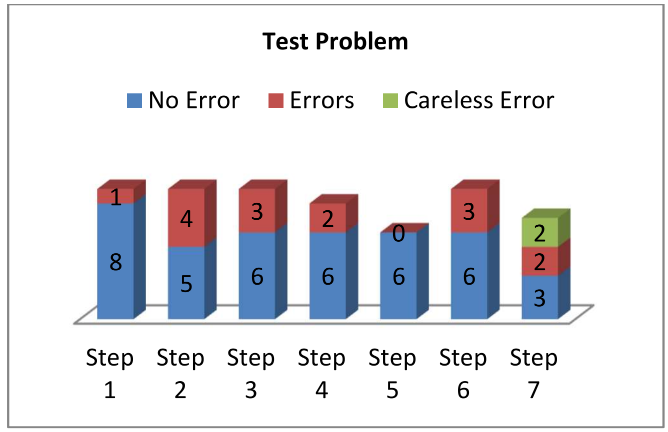

Figure 2: Errors in Test Problem

Although making errors in one step could bring difficulties in completing the next steps, the mathematical concept of each step may not correlate with each other. In addition, in the test problem students made fewer errors than on the quiz.

Misconceptions may affect students' confidence levels as well as their performance. Students were asked to designate their confidence level for every problem, which was averaged later to find the overall confidence rating (see Table 11). Professor Han provided the performance rating for each student. Usually, the confidence level was lower than the expected performance level. On the quiz, however, the confidence rating was slightly higher than the performance rating for the students who made errors.

Table 11. The Confidence Rating and Performance Rating on a Quiz Question and a Test Question

	Quiz		Test	
Students	**Average confidence rating**	**Average performance rating**	**Average confidence rating**	**Average performance rating**
Who did not make errors	**4.429**	**5**	**3.692**	**4.923**
Who made errors	**2.444**	**2**	**2.333**	**2.889**

<u>Discussion</u>

Results from the Spring 2014 questionnaires suggest that understanding and finding interest in mathematics influence students' learning. Students mentioned different ways they try to solve a problem (Question #4) including: (1) identify the missing information, (2) think logically, (3) follow the steps to fit formulas, (4) practice, (5) draw diagram,(6) look at class-notes, and (7) ask for help. Darry (1996) mentioned that the learner needs *scaffolding* to learn new concepts, and the students used various techniques to learn Mathematics concepts. Furthermore, students were aware of the importance of the Math course (Question #3), and some students also mentioned the real-life applications of the course (Hasan, 2001).

It is important to categorize the problems that are performed comfortably by students alone, the problems that are completed with less assistance, and the problems that stump the students (Questions #5, #6 & #8). Zack (1999) suggests that mathematics could be more interesting if it involves students with local culture. Students can ask for help from a professor or any person who knows the subject, in workshop, or from class notes (Question #7). Interestingly, no student referred to the assistance of the textbook, despite this being the erstwhile standard and convenient source to build mathematics concepts. Students could garner the learning from external sources to internalize it by themselves or with assistance (Question #9).

The observations from the quizzes and test papers might suggest that the students who solved the problems correctly only followed a certain algorithm (Derry, 2013).The misconception on step two led students in making further errors, so they might know the steps but did not get the right answer(Egodawatte, 2011).Students could not follow the process that the professor previously showed in the class, possibly because of misconceptions which interfere with learning (Smith et al., 1993-1994).Students had trouble with the negative sign and made careless mistakes. Koedinger, et al. (2008) found that the students who have misconceptions overlap rules and formulas, and in this study, we found that students became confused with different mathematics rules while solving one problem. The anxiety provoked by tests could impact students' errors, given students' lower performance level in the test. Also, the lower confidence rating that students reported on the test which suggests that students were more nervous on the test than on the quiz.

Conclusions

Students try to construct their new knowledge using their prior understanding. Therefore, in constructing concepts for a college first-level mathematics course, students will use their high school math knowledge (Roth et al., 2001). The workshop environment assists students in moving from the bottom of the ZDP towards the top. There is no restriction to what is used as assistance to move up the ZDP and students can use anything as support to build new concepts. As a Peer Leader my role is to be a part of their assistance for learning and provide support as scaffolding (Daniels,1996).

However, undeveloped concepts may be in part the reason for the misconceptions, which led students to make errors. Further research should examine how misconceptions can be reduced. Open discussion about how math concepts are applied may help students become more interested in mathematics – a suggestion for Peer Leaders.

This type of research helps support students in workshop by helping peer leaders understand the general learning process, the learning interference, and the reasoning behind making major and minor errors. Ultimately Peer Leaders who understand how to ease the student's cognitive leaning process will provide a better learning environment in the PLTL workshops.

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